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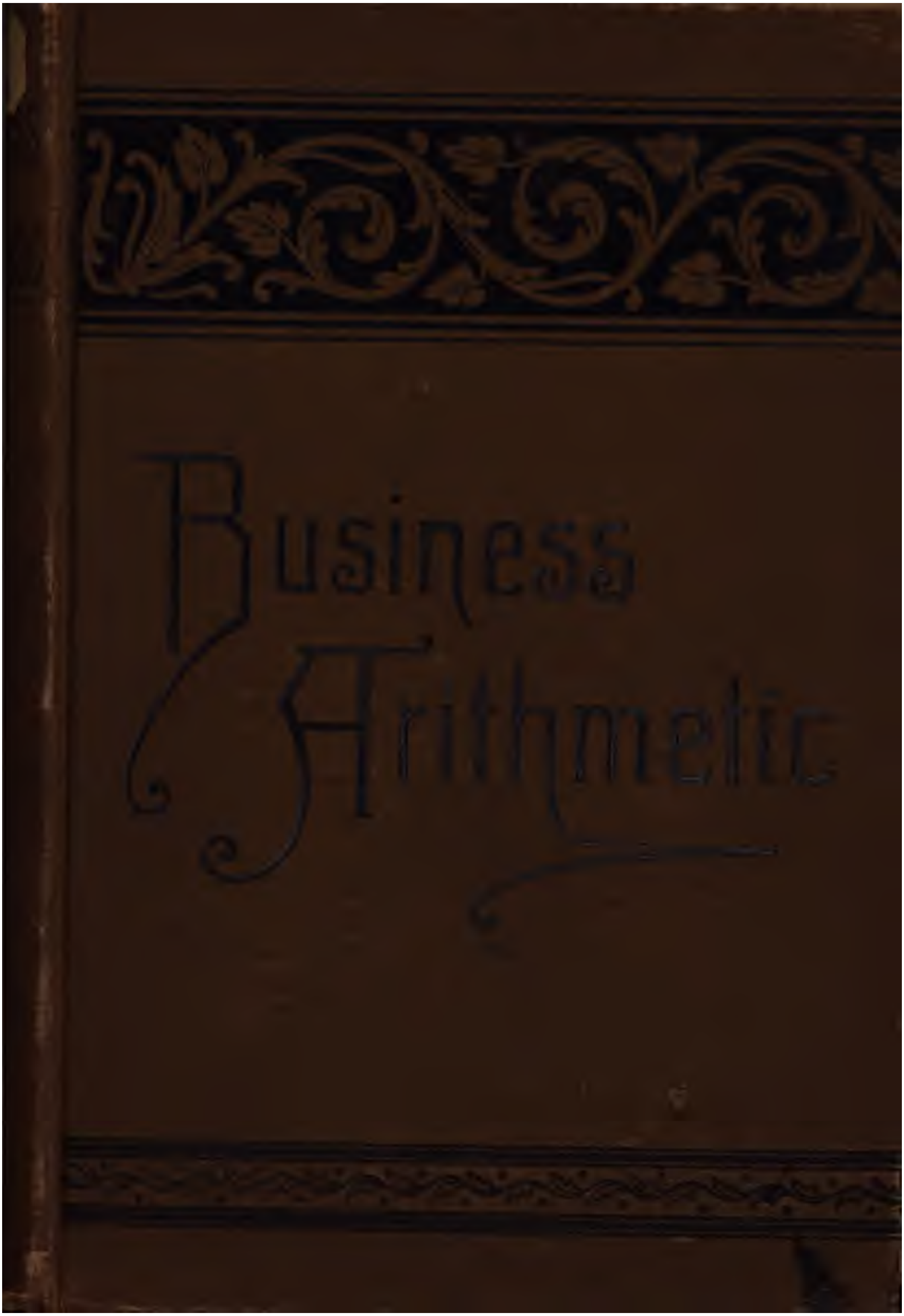

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BUSINESS

ARITHMETIC

DESIGNED FOR USE IN ALL SCHOOLS IN WHICH THE

COMMERCIAL BRANCHES ARE TAUGHT

AND AS A

BOOK OF REFERENCE

FOR

BUSINESS MEN

1893.

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PREFACE.

A text book on Business Arithmetic should embrace all of the requirements of commercial calculations. It should not only contain the theories and principles which underlie arithmetical computations, but it should aim to render the pupil expert in applying them to practical work. The real test of the value of the pupil's knowledge of this subject is the facility with which he can perform his operations. Accuracy and rapidity are absolute essentials in business.

In preparing this book two things have been constantly kept in mind: the introduction of such subjects as will provide a thorough knowledge of the requirements of business in this direction, and to present the topics in a teachable way. Unimportant subjects have been omitted, and arithmetical puzzles have been carefully avoided.

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BUSINESS ARITHMETIC.

DEFINITIONS.

1. **Arithmetic** is the science of numbers and the art of computation.

Science is a classification of facts, and as applied to Arithmetic it deals with the principles, properties, and relations of numbers. Art is the power of applying a system of knowledge, and as applied to Arithmetic, it is the power of applying the knowledge of numbers to practical purposes.

2. **Number** expresses an idea of how many, and refers to a single thing or to a collection of single things.

3. **A Unit** is a single thing. It is the least whole number.

A unit is the object, the thing itself, while a number conveys the idea of how many units are considered. Thus, an apple in a fruit dish is a unit, and seven apples is a collection of units. But to refer to apples by figures the number indicates how many; as, 7 apples.

4. The **Unit of a Number** is one of the single things forming the collection represented by the number. Thus, the unit of 5 is 1; of 13 dollars, 1 dollar; of 18 boys, 1 boy.

5. **Units of Computation** are the standards by which we compute. Thus, the unit of eggs sold by the dozen is 1 dozen; of lumber, sold by the hundred, 1 hundred; by the thousand, 1 thousand, etc.

6. **A Concrete Number** refers to some particular thing or quantity, as 7 apples, 18 boys, 13 dollars, 5 bushels of wheat.

7. **An Abstract Number** refers to no particular thing, but indicates unity or a certain number of times unity; as 1, 3, 5, 50, &c.

Usually, in computations, abstract numbers represent a repetition of some particular thing understood, which in itself properly expressed would be a concrete number; but 1 standing alone is understood as unity and an abstract number.

NOTATION AND NUMERATION.

8. **Notation** is the art of writing or expressing numbers by means of words or characters. There are two methods of notation in use, the *Arabic* and the *Roman*.

9. **Numeration** is the art of reading numbers expressed by characters.

10. The **Arabic Notation** is the method of expressing numbers by figures. This method employs *ten* different figures to represent numbers, viz.:

0,	*1,*	*2,*	*3,*	*4,*	*5,*	*6,*	*7,*	*8,*	*9.*
Naught,	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine.

Figures are arbitrary characters used to express numbers. The first character is called *Naught*, *Cipher*, or *Zero*, because when standing alone it has no value. The other nine characters are called *significant figures*, because each, when standing alone, signifies something or has a value of its own. These nine figures are also called *digits*. By combining these ten figures according to certain principles, any number may be expressed.

11. By the Arabic Notation the value of numbers increases from right to left and decreases from left to right in a ten-fold ratio; the successive figures from right to left or from left to right are called *orders of units*, the value of one of any order being ten times the value of one of the order next on its right, and only one-tenth of the value of the order next on its left.

12. This method of numeration establishes a decimal system in which the successive orders are grouped for convenience into periods of three figures each as shown in the following

NUMERATION TABLE.

Periods.	6th			5th			4th			3d			2d			1st		
Name.	Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
Orders of units in the Periods.	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
Number.		3	7,	4	0	6,	5	2	3,	0	7	1,	3	0	0,	3	8	4

13. The Arabic method of notation is based upon the following

General Principles.—*1. The removal of any figure one place toward the left multiplies its value by ten; two places, by one hundred; three places by one thousand, etc.*

2. The removal of any figure one place toward the right divides its value by ten; two places, by one hundred, etc.

3. A cipher placed after a significant figure multiplies it by ten; two ciphers so placed multiplies it by one hundred, etc.

14. To read numbers expressed by figures.

Rule.—*Separate the numbers into periods of three figures each, read each period as though it stood alone, then add the name of the period.*

15. The following table will serve to illustrate the application of the above principles and rule.

COMBINED NOTATION AND NUMERATION TABLE.

Illustrative Examples.	Periods.	Millions			Thousands			Units		
	Orders.	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
1. Write One hundred twenty-five *millions*, one hundred twenty-five *thousands*, one hundred twenty-five *units*.		1	2	5	1	2	5	1	2	5
2. Write One hundred *millions*, one hundred *thousands*, one hundred *units*.		1	0	0	1	0	0	1	0	0
3. Write Ten *millions*, ten *thousands*, ten *units*.			1	0	0	1	0	0	1	0
4. Write One *million*, one *thousand*, one *unit*.				1	0	0	1	0	0	1
5. Write Five *millions*, three hundred six *units*.				5	0	0	0	3	0	6

16. Enumerate and read each one of the above numbers.

In example 2 observe that 1 *hundred* of millions = 100 *ones* of millions; that 1 *hundred* of thousands = 100 *ones* of thousands; and that 1 *hundred* of units = 100 *ones* of units.

EXAMPLES.

17. See illustrative examples, p. 3.

1. Write 3 *hundreds,* 1 *tens,* 2 *ones of Millions,* 3 *hundreds,* 1 *tens* 2 *ones of Thousands,* 3 *hundreds,* 1 *tens,* 2 *ones of Units.*

2. Write 312 *Millions,* 312 *Thousands,* 312 *Units.*

3. Write Four hundred *Millions,* four hundred *Thousands,* four hundred *Units.*

4. Write Eight *Millions,* seven hundred eight *Units.*

5. Write 8 *ones of Millions,* 0 *hundreds,* 0 *tens,* 0 *ones of Thousands,* 7 *hundreds,* 0 *tens,* 8 *ones of Units.*

18. Express by figures the following numbers:

1. Forty-four.
2. Three hundred seven.
3. One thousand twenty-one.
4. Thirteen hundred seventy-six.
5. Fifty-nine thousand one hundred nine.
6. Seventeen thousand seven hundred seventy.
7. One hundred five thousand ninety-seven.
8. Five million fourteen thousand three hundred five.
9. Forty-one billion seven hundred eighty-one million twenty-one.
10. One hundred fifteen billion two hundred seventeen million five hundred seven thousand six hundred ninety-one.

19. Copy, point off and read the following numbers.

1. 386.
2. 1341.
3. 12406.
4. 872463.
5. 4681005.
6. 200020.
7. 5860092.
8. 453763075.
9. 1210038203.
10. 894076403010.

20. Write in words and read the following numbers:

1. 37.
2. 154.
3. 7096.
4. 13456.
5. 25487.
6. 547.
7. 8926.
8. 10010.
9. 5325036.
10. 97680405.

21. The **Roman Notation** is the method of expressing numbers by letters. This method employs seven capital letters to express numbers, viz.:

I	V	X	L	C	D	M
One,	Five,	Ten,	Fifty,	One Hundred,	Five Hundred,	One Thousand.

22. The method of using these letters to denote numbers is shown in the following

COMPARATIVE NOTATION TABLE.

Roman.	Words.	Arabic.	Roman.	Words.	Arabic.	Roman.	Words.	Arabic.
I	*One*	1	XI	*Eleven*	11	XXX	*Thirty*	30
II	*Two*	2	XII	*Twelve*	12	XL	*Forty*	40
III	*Three*	3	XIII	*Thirteen*	13	L	*Fifty*	50
IV	*Four*	4	XIV	*Fourteen*	14	LX	*Sixty*	60
V	*Five*	5	XV	*Fifteen*	15	LXX	*Seventy*	70
VI	*Six*	6	XVI	*Sixteen*	16	LXXX	*Eighty*	80
VII	*Seven*	7	XVII	*Seventeen*	17	XC	*Ninety*	90
VIII	*Eight*	8	XVIII	*Eighteen*	18	C	*One Hundred*	100
IX	*Nine*	9	XIX	*Nineteen*	19	D	*Five Hundred*	500
X	*Ten*	10	XX	*Twenty*	20	M	*One Thousand*	1000

Repeating a letter, as seen by the above table, repeats the value of the letter. A letter of *less* value placed *before* one of *greater* value diminishes the value of the greater; and a letter of *less* value placed *after* one of greater value increases the value of the greater. Placing a dash over a letter increases its value one thousand times; as, V for 5, $\overline{\text{V}}$ for five thousand.

EXAMPLES.

23. Express by Roman notation:

1. Twenty-four.
2. Fifty-seven.
3. Eighty-eight.
4. Ninety-six.
5. One hundred eleven.
6. Six hundred fifty-four.
7. Nineteen hundred nineteen.
8. Forty thousand sixty-four.
9. 175.
10. 2196.
11. 75354.
12. 998649.

24. Express by Arabic notation:

1. XCII.
2. XXVII.
3. XXIX.
4. CLX.
5. MCXIX.
6. MCLXIX.
7. MDCXCII.
8. MDCCLVI.
9. $\overline{\text{DCV}}$.
10. $\overline{\text{DLXX}}$.
11. $\overline{\text{MDC}}$.
12. $\overline{\text{DCCXLV}}$.

ADDITION.

25. **Addition** is the process of finding the sum of two or more given numbers.

26. The **Sum or Amount** is the result obtained, or the number found.

27. The **Sign of Addition** is +, and it is called *plus*, which signifies more. When placed between two or more numbers, or when several numbers are connected by this sign, it signifies that their sum is to be found; as, 5 + 3 + 4, is read 5 plus 3 plus 4, and means that their sum, which is 12, is required.

28. The **Sign of Equality** is =, and is read *equals*, or is *equal to*. When placed between two numbers or combinations of numbers it indicates that their values are the same; as, 5 + 3 + 4 = 12, read 5 plus 3 plus 4 equals 12, and means that the value of 12 equals the value of the numbers at the left of the sign of equality.

29. The sign of dollars is $; of cents is ¢, or ct., or cts. Dollars and cents when written together are separated by a period, with the dollar sign used only and preceding the amount; as, $15.48, read fifteen dollars and forty-eight cents.

30. The double ex, # when placed *before* a figure is read *number*. When placed *after* a figure it is read *pounds*; as, #8 is read *number eight*, 8# is read *eight pounds*.

31. The *printed* figures used in expressing numbers differ from the *written* ones, and the latter should be practiced by the student until he can make them correctly and of proper size.

32. **Principles.**—*1. Only like numbers and like unit orders can be added one to another.*

2. The sum or amount contains as many units as all the numbers added.

3. The sum or amount is the same in whatever order the numbers be added.

33. Numbers are written for addition either in *vertical* or *horizontal* order.

34. There are three methods of addition in common use, viz.: the *Elementary method,* the *Result method,* and the *Group or Reading method.*

The student is supposed to be familiar with the first one of these methods, and hence it will not be treated here. Daily drill in the third method is urgently advised with all pupils during the entire period of their study of arithmetic. Too much importance can scarcely be attached to this suggestion.

35. The result method of addition.

EXAMPLE.—Add 398, 269, 125, and 143.

OPERATION.

$$\begin{array}{r} 398 \\ 269 \\ 125 \\ 143 \\ \hline 935 \\ {\scriptstyle 2\,2} \end{array}$$

EXPLANATION.—Arrange the numbers in vertical columns so that units of like orders stand in the same column. Commence with the right-hand or one's column and add, "*three, eight, seventeen, twenty-five.*" Twenty-five *ones* are equal to 2 *tens* and 5 *ones.* Write the 5 ones beneath the column of ones added and add the 2 tens to the column of tens. Commencing with the carrying figure 2, which is indicated in small type under the tens column, in the operation, add, "*two, six, eight, fourteen, twenty-three.*" Twenty-three tens are equal to 2 hundreds and 3 tens. Write the 3 tens beneath the tens column and add the 2 hundreds to the column of hundreds, placing the carrying figure below the hundreds column, as above explained. Commencing with the 2 hundred (carried) add "*two, three, four, six, nine.*" Write the 9 hundred thus obtained beneath the hundreds column, and the answer is 935.

Prove by adding the columns downwards; if the two results agree, the work is presumed to be correct.

Rule.—I. *Write the numbers so that figures of the same order stand in the same column.*

II. *Commencing at the right, add each column separately, and write the sum, if expressed by one figure, under the column added.*

III. *If the sum of any column consists of more than one figure, write the right-hand figure under the column added, and add the remaining figure or figures to the next column.*

Proof.—*Add the columns in the opposite direction, and if the two results agree the work is probably correct.*

To preserve the carrying figures various methods are employed. When the columns are footed "in pencil" it is customary to write the carrying figures below the footing, as illustrated above. When the columns are footed in ink, and especially in books of account and in other business documents, the carrying figures are often written on a separate piece of paper. Sometimes the footings are also so written until they are proven to be correct, when they are then written in their proper places. Again many accountants, and others, practice the memorizing of the carrying figures. The student is advised to adopt the latter method.

The ability to add long columns of figures accurately and rapidly is acquired only by constant, faithful practice; but it is worth many times its cost, and it should be the ambition of every student to attain it. Practice first for *accuracy*, and then for rapidity and accuracy. Both are essential to success.

EXAMPLES.

36. Copy and add the following :

(1)	(2)	(3)	(4)	(5)	(6)
853	226	986	398	375	718
964	314	217	712	865	952
827	916	324	354	734	716
346	244	622	666	854	298
933	397	393	298	923	754
692	853	787	854	689	283
542	614	286	642	564	647
386	813	942	926	436	954

(7)	(8)	(9)	(10)	(11)	(12)
3246	7986	3296	9528	2296	8529
8973	6897	5835	7282	9529	9843
5498	4789	9463	6493	3824	6687
7562	9874	2736	7592	9842	7963
9364	3758	9542	8678	6282	3964
2798	7583	2954	2943	4798	4976
8642	2969	9647	6386	8364	8437
4283	9392	8362	2954	2978	2796

(13)	(14)	(15)	(16)	(17)	(18)
23119	98969	89426	88268	69825	94869
86392	82623	28769	23982	32758	28942
94286	28925	32649	29876	28363	68394
29986	87639	86426	69843	98678	95832
39628	28762	38789	29869	39289	38964
98694	94384	92836	78342	75138	27968

(19)	(20)	(21)	(22)	(23)	(24)
42869	32698	395863	98625	98664	93628
1807	8794	284006	8309	29678	9642
10360	836	189	86	1693	179
29	1494	6875	92387	82029	642
614	78	394	8962	3269	8623
8296	346	186309	90863	18939	2975
23694	753	273091	28967	29393	92839
6986	6885	854	52876	8295	26954
96	618	914	8934	36968	32869
9	10	183	16753	986	1948
14360	16798	286798	92189	31293	32692

37. The Group or Reading Method of Addition.—This method consists of grouping or reading two or more figures in a column, and treating them as a part of the result, the same as several letters are grouped and taken together to form one word in a sentence. In reading we do not usually stop to spell the words, but we see all of the letters in each word, and hence the whole word, at first sight. The same should be done in adding columns of figures. Each figure should be treated as a letter, each group as a word, and each column as a sentence. A little patient practice is all that is necessary to acquire skill in the use of this method.

In practicing the above method of addition it will be of great assistance to the student if he previously commit to memory the combinations forming ten or double ten. Following are some of these combinations:

	9	8	7	6	5				
By two figures	1	2	3	4	5				
	8	7	6	6	5	5	4	4	
	1	2	3	2	4	3	4	3	
By three figures	1	1	1	2	1	2	2	3	
	8	9	9	7	6	8	8	9	9
	8	8	9	7	6	7	3	7	6
Double tens	4	3	2	6	8	5	9	4	5

Other combinations should be formed and studied until the student is thoroughly familiar with them and able to use them in his daily work.

38. To add by the reading method.

EXAMPLE 1.—Add 9, 6, 5, 7, 3, 8, 3, 9, 9, 1.

OPERATION.

$$\begin{array}{ll}
d = \text{double ten} & \left\{\begin{array}{r}9\\6\\5\end{array}\right. \\
c = \text{ten} & \left\{\begin{array}{r}7\\3\end{array}\right. \\
b = \text{double ten} & \left\{\begin{array}{r}8\\3\\9\end{array}\right. \\
a = \text{ten} & \left\{\begin{array}{r}9\\1\end{array}\right. \\ \hline
 & \;\;60
\end{array}$$

EXPLANATION.—Beginning at the bottom add by tens and double tens in the order indicated by the letters a, b, c, etc., and say *ten, thirty, forty, sixty.*

EXAMPLE 2.—Add 3, 8, 7, 5, 6, 9, 4, 1, 6, 6, 8.

OPERATION.

$$\begin{array}{lr}
e & 3 \\
d & \left\{\begin{array}{r}8\\7\\5\end{array}\right. \\
c & 6 \\
b & 9 \\
c & 4 \\
b & 1 \\
a & \left\{\begin{array}{r}6\\6\\8\end{array}\right. \\ \hline
 & 63
\end{array}$$

EXPLANATION.—In this example the groups do not occur in consecutive order, and in some instances the figures forming a group are in different places in the column. Those marked *a* are a part of one group and those marked *b*, of another, etc. Beginning with *a* add by tens and double tens as follows: *twenty* (a = 8, 6, 6), *thirty* (b = 1, 9), *forty* (c = 4, 6), *sixty* (d = 5, 7, 8), *sixty-three* (e = 3).

39. Short Cuts in Grouping.—In rapid addition performed by the grouping method, it is often convenient to drop a figure or to borrow a figure for the purpose of completing a group. The difference caused by dropping or borrowing is restored at a place in the column where such restoration would just complete a new group.

Sometimes an odd figure occurs between two groups, and with the rapidity of thought the accountant will skip such figure while he continues to add the other figures in the column; but, keeping this omitted figure in mind, he will either combine it with others to form a new group or he will add it at the end of the column.

ILLUSTRATIVE EXAMPLES.

EXAMPLE 1.—Add 3, 5, 5, 6, 8, 6, 9, 6, 4, 5, 6.

EXPLANATION.—Instead of saying *six, eleven, twenty-one,* etc., say *ten* (6 + 5—the one dropped) *twenty* (4 + 6) *thirty* (9 + the *one* dropped) *fifty* (6 + 8 + 6) *sixty* (5 + 5) *sixty three.*

EXAMPLE 2.—Add 7, 5, 5, 2, 8, 2, 7.

EXPLANATION.—Enclosing the figures in parentheses to indicate the groups gives (7 + 2) + (8 + 2) + (5 + 5) + 7, and read *ten* (one short) *twenty, thirty, thirty-six* (shortage of 1 restored in adding the 7).

EXAMPLE 3.—Add 3, 1, 9, 6, 5, 7, 8, 3, 7.

EXPLANATION.—Enclosing the groups we have (7 + 3) + (8 + 7 + 5) [skip 6] + (9 + 1) + (3 and the 6 skipped) and they are read *ten, thirty, forty, forty-nine.*

EXAMPLES.

40. Re-write each of the following examples, arranging the figures in perpendicular columns and add by the reading method. Prove by reading the columns in the reverse order. In writing the figures, take pains to make them small and legible.

Keep the point of your pencil sharp and it will aid you greatly in making good figures.

1. 37, 55, 13, 32, 87, 15, 25, 76, 61, 53, 42, 38.

2. 18, 82, 47, 21, 51, 103, 82, 74, 65, 43, 35, 51, 16.

3. 93, 72, 28, 45, 52, 83, 48, 78, 82, 60, 85, 36, 27, 81.

4. 361, 531, 455, 322, 119, 334, 279, 191, 253, 219.

5. 290, 200, 815, 954, 723, 521, 279, 808, 141.

6. 2, 182, 44, 818, 289, 19, 7, 41, 826, 28, 1, 897, 10.

7. 36, 656, 92, 4, 27, 221, 64, 783, 43, 706, 825, 3, 19.

8. 120, 45, 8, 16, 119, 33, 8, 650, 1103, 36, 78, 114, 6, 149, 85, 3012.

9. 3680, 153, 23, 1784, 9, 413, 70, 1815, 2056.

10. 15040, 34780, 10075, 16550, 188178, 63310, 36316, 22370, 19910, 10150, 9130, 21370, 6120, 9900, 44039, 10120, 10700.

11. A farmer sells a commission merchant ten loads of Early Rose potatoes which are scored in the farmer's pass book as follows ;—38, 43, 29, 41, 37, 35, 33, 28, 46, 40 bush. each, respectively. How many bushels in all ?

12. The receiving clerk of a wholesale grocery store checks off ten brls. of "A" sugar weighing as follows : 318 #, 315 #, 317 #, 322 #, 298 #, 326 #, 322 #, 328 #, 308 #, 311 #. How many pounds of sugar in the invoice? (See art. 30, page 6).

13. Walter Merrell sold E. M. Morgan on acct. 10 loads of turnips which measured out in bushels as follows: 38, 36, 43, 54, 39, 46, 37, 47, 39, 46. How many bushels in the ten loads?

14. For the month of November there registered in a certain city at the Y. M. C. A. visitors for each consecutive day of the month as follows: 815, 124, 89, 64, 47, 289, 978, 60, 46, 182, 76, 89, 317, 1054, 98, 129, 92, 73, 54, 321, 825, 88, 49, 62, 85, 998, 27, 62, 85, 1075. How many persons registered during the month?

15. Green, Morgan & Co. bought of The Nonatuck Manufacturing Co., Providence, 10 bales of Jersey cloth measuring in yards as follows: 198, 176, 154, 211, 149, 198, 176, 188, 144, 154. How many yards were there in the purchase?

16. The land surfaces of the District of Columbia, the territories and the new states are as follows in sq. miles: Alaska 531409, Arizona 112920, the Dakotas 147700, District of Columbia 60, Idaho 84290, Indian territory 64090, Montana 145310, New Mexico 122460, Utah 82190, Washington 66880, Wyoming 97575, Public lands unattached 5740. How many square miles of land in all?

41. Adding in Money Columns.—A large part of the errors that occur in business are in simple addition, and many of these errors are due to the figures being placed irregularly in money columns. One of the first difficulties that a young man will encounter when he enters business, unless he has had thorough preparation, will be his inability to add accurately long money columns. One of the principal reasons for this is that instead of solid columns of figures, and numbers of uniform width, he finds many different amounts, varying in number of figures from two in the cent columns to six or eight in the dollar columns. These figures are written on lines wide apart, and often in irregular order, which makes it difficult for any one not accustomed to such work, to add the figures of any one order and follow them to the top of the column.

To overcome this difficulty observe the following:

After adding the cent columns, which are not usually very irregular, commence with the first column at the left of the dividing line, and in adding this think of and look for the *first* figure in each amount as you go up the column. This figure will sometimes be found near the line, and sometimes under the tens or even hundreds figure of the number above. In adding the next or tens column think of and look for the second figure from the line; for hundreds, the third figure, etc. In this way long and irregular columns of figures may be accurately and rapidly added.

EXAMPLES.

42. Copy and add the following:

(*1*)		(*2*)		(*3*)		(*4*)		(*5*)	
6132	85	100	25	5	25	500	26	500	82
168	53	10	83	18	13	32	18	18	18
17	22	25	07	7	50	1900		1837	54
186	19	118	52	300		38	54	2	86
5	54	164	75	1409	17	300	12	14	95
16	28	317	63	58	19	90	53	111	
5		209	29	500	28	26	75	513	45
1870	09	152	01	300	24	8133		1234	56
1	25	317	08	1	09	1	08	2109	
500		1800	75		38	500	75	1	50
	82	150	83	1008		75		13	18
100	17	1	25	1	54	84	62	1	54
3	23	90	52	87	63	185	23	62	
1600		1	86	1	29	62	82	49	54
10	25	500	76	19	28	7119	18	16	27

6. 34318.01, 4289.15, 188., 23797.07, 13037.25, 1240.52, 3409.81, 477.10, .36, 30.50, 1200.00.

7. 3224.42, 80.00, 1209.15, 17903.80, 4.50, 1992.58, 56.75, 495.10, 29.50, .02.

8. 1319.20, 60.31, 8487.05, 5000., 520.56, 4797., 13., 88.05, 658784.39, 4059.10, 784.62, 145179.09, 1791.05.

9. 1576.03, 114220.35, 97.62, 2125.05, 272.03, 47315.50, 20.21, .53, 114791.25, 3709.50, .10.

10. 563465.18, 60719.05, 359., 1.43, 164132.09, 231.09, 165762.51, 6.24, 10129.39, .05, 876.19.

The student is advised to spend ten or fifteen minutes daily in practicing rapid addition. Examples for this purpose may be taken from any statistical work, or they may be originated by the student himself. Odd

moments can be very profitably spent in this way. Let the teacher insist upon small figures, regular columns, legible record, and as a result accuracy and rapidity will follow.

HORIZONTAL ADDITION.

43. Horizontal Addition is the process of adding numbers from right to left or from left to right when they are written in horizontal lines, without arranging them in vertical columns.

In adding numbers written horizontally, more care is required that the units added shall be of like order, and greater certainty of correctness can be had by adding first from right to left and then from left to right.

The group or reading method may be employed with equal advantage where numbers are written horizontally.

44. Fourths and Eighths.—In recording measurements of cloth and some other articles of merchandise, fourths and eighths are written at the right and a little above the whole number; thus 18^3, 17^2, 14^1, are read *eighteen and three-fourths, seventeen and two-fourths*, and *fourteen and one-fourth,* when the parts are fourths. Sometimes the parts are eighths, and then 17^2 would be read *seventeen and two-eighths.*

To the student who has no knowledge of fractions such work as the above will not be difficult if he will remember that each 25 cent piece is known as 1 fourth ($\frac{1}{4}$) of a dollar; that 4 fourths ($\frac{4}{4}$) of a dollar are equal to 1 dollar, and that 6 fourths ($\frac{6}{4}$) are equal to one and 2 fourths ($1\frac{2}{4}$) dollars. The same applies to fourths in general. Four-fourths ($\frac{4}{4}$) of a yard equal 1 yard; eight-eighths of a gallon equal 1 gallon, etc. When the denominator, the number below the line, is not given or mentioned, *fourths* are understood.

45. To add horizontally.

EXAMPLE.—Add 18^3, 14^1, 17^2.

OPERATION.

$18^3, 14^1, 17^2 = 50^2$

EXPLANATION.—Beginning with the small figures which indicate fourths, say, "*two, three, six, six-fourths*," which equal 1 and 2 fourths. Write the 2 fourths at the right of the given numbers and a little above the line and add the one to the first right-hand figure of the whole numbers. Then say, "*eight, twelve, twenty*." Write the cipher at the right and on the line, close to the fractional part of the result. Next carry the 2 tens to the first tens figure and say "*three, four, five.*" Write the 5 on the line and at the left of the other figures in the result, and the work is complete. The sum of the figures given is found to be Fifty and two-fourths.

Prove this result by adding the figures from left to right.

EXAMPLES.

46. Copy and add from right to left, and review from left to right.

1. 5, 3, 6, 1, 8, 2, 9, 3, 7, 4, 2, 5, 8, 7, 8, 2, 3, 5, 4.
2. 15, 23, 36, 18, 25, 53, 92, 20, 56, 12, 10, 37.
3. 66, 29, 5, 14, 71, 30, 2, 13, 40, 6, 73, 10, 5, 20.
4. 510, 297, 69, 841, 50, 203, 7, 28, 1410, 9, 315.
5. 315, 405, 125, 96, 856, 31, 76, 903, 1215, 2050, 1891, 583.
6. 16^3, 14^1, 76, 84^2, 54^2, 18^1, 19, 38, 54^2, 18^3, 28^1, 76^3, 88^1, 23^3.

The sum of the fourths is 22. $22 \div 4 = 5^2$, or five and two-fourths

7. 37^1, 32^3, 33, 35^3, 34^1, 32, 35^2, 33^3, 37, 38^1, 48^1, 36.
8. 41^2, 46^3, 41^1, 45^2, 44, 44^1, 47^1, 45^3, 42, 42^3, 43^3, 43^1, 47.
9. 55, 45^3, 55^2, 50^3, 51, 52, 46^1, 50, 52^1, 54, 48^2, 50^3, 52, 55^1.
10. 45, 48^1, 46^1, 34^2, 45^3, 44^3, 46, 44, 48, 46, 42, 50^2, 51^3, 46^2, 47^1, 46^1, 48, 49, 45^1, 48.

Horizontal addition may sometimes be employed to advantage in adding dollars and cents; in such cases it is best to omit the dollar sign; as, for $5.25, write 5.25.

47. Copy and add horizontally, and prove results:

1. 5.25, 8.17, 11.40, 1.82, 16.02, 90.70, 146.24, 9.11.
2. 210.10, 46.98, 5.50, 108.12, 4.75, 26.53, 92.00.
3. 5.71, 108.97, 29.33, 150.09, 46.07, 1.95, 231.45.
4. 278.19, 105.29, 80.50, 19.93, 25.01, 1.13, 17.56.
5. 5.40, 88.45, 90.63, 112.50, 45.95, 87.08, 306.82.

48. Calling Off and Rechecking.—In calling off examples for practice, in reporting sales or inventories to an entry clerk, or in rechecking schedules in dollars and cents, great care should be taken and the one right method used. In calling the following amounts, $500.18, $18.82, $1837.54, $2.86, call as follows: Five hundred *dollars* eighteen; eighteen *dollars* eighty-two; eighteen hundred thirty-seven *dollars* fifty-four; two *dollars* eighty-six.

Observe that the word dollars is called with each amount. It is unneccessary to call the word cents, but the word dollars should never be omitted. Should the last amount above be called "Two eighty-six," the way one often hears such amounts called, it might be recorded $286. Any amount of four figures should be called in hundreds and not in thousands. The amount $1837.54 above should not be called *One thousand eight hundred, etc.*, but *eighteen hundred, etc.* This should also be observed in writing out amounts in notes, drafts or other business documents.

SUBTRACTION.

49. Subtraction is the process of finding the difference between two numbers.

50. The Difference or Remainder is the result obtained by subtracting one number from another.

51. The Sign of Subtraction is a short horizontal line, —. It is called *minus,* which signifies *less.*

When the sign of subtraction is placed between two numbers, as, 8 — 2, it indicates that the larger number is to be diminished by as many units as there are units in the smaller number.

The larger number is called the *minuend,* and the smaller number, the *subtrahend.*

52. Principles.—*1. Only like numbers and units of the same order can be subtracted, one from the other.*

2. The sum of the subtrahend and the remainder must be equal to the minuend.

53. To find the difference between two numbers.

EXAMPLE.—A man invested $859 in potatoes, which he sold for $1236. How much did he gain?

OPERATION.

$1236 Minuend.
859 Subtrahend.
$377 Remainder.

EXPLANATION.—Write the subtrahend, the number to be subtracted, under the minuend, the number to be diminished, with units of the same order under each other. Commence with ones, and, as 9 ones cannot be taken from 6 ones, take 1 ten from 3 tens and add it to the 6 ones. 1 ten + 6 ones = 16 ones. 9 ones from 16 ones leaves 7 ones. Write down the 7 ones. As 5 tens cannot be taken from 2 tens (3--1) change one hundred to tens and add to the 2 tens. 1 hundred + 2 tens = 12 tens. 5 tens from 12 tens leaves 7 tens. Write down the 7 tens and subtract 8 hundred from 11 hundred, which leaves 3 hundred. Hence the answer, 3 hundreds, 7 tens, and 7 ones, or 377 ones, or $377.

This process is called "borrowing tens." Having mastered the theory, the ordinary and most convenient method for practice is to leave the minuend in its original form, and, when borrowing is necessary, add 1 to the succeeding subtrahend figure.

Rule.—I. *Write the subtrahend under the minuend so that units of the same order stand in the same vertical line.*

II. *Begin at the right and subtract each figure of the subtrahend from the corresponding figure of the minuend. When necessary borrow ten and mentally add one to the next subtrahend figure.*

Proof.—*Add the remainder to the subtrahend; if the sum equals the minuend the work is correct.*

EXAMPLES.

54. Find the results of

1. 484 — 231.
2. 531 — 362.
3. 870 — 396.
4. 504 — 148.
5. 781 — 695.
6. 7873 — 2416.
7. 8645 — 2347.
8. 7048 — 5063.
9. 6949 — 3237.
10. 9000 — 1999.
11. 13752 — 10685.
12. 20907 — 19898.
13. 33450 — 29805.
14. 20000 — 12345.
15. 23456 — 14567.

16. A grocer bought 861 lbs. of sugar. He sold 349 lbs. to Brown and the remainder to Clark. How many pounds did Clark buy?

17. After drawing $317 from bank, I had $517 left to my credit. How many dollars did I have on deposit at first?

The proof of subtraction is of as much importance to learn as the process itself, and every example should be subjected to the proof, for in this way only can absolute accuracy be secured.

18. How many times can 25 be taken from 125?

When the numbers are equal as in the above example, a shorter method is used than the one by subtraction, see page 25.

19. I bought 23,240 acres of Dakota land, and sold at times 1000, 320, 520, 640, 3200, 2520, 160, and 1920 acres. How many acres had I remaining?

20. A man bought a farm for $3250. He built a house on it at a cost of $1850, fences costing $416, and then sold it for $7500. What was his gain?

21. My book-keeper's salary is $1450 per year. If he requires for his rent, $365; for personal expenses, $170; and for support of his family, $775; what amount will he have left at the end of the year?

22. The cost of my lot was $1750. I paid for mason work on my house, $1210; for carpenter work, $5145; for plumbing, $985; for decorating, $1650; for painting, $625; for grading, sodding, and fencing grounds, $590. The interest on outlays to date of sale was $315. I then sold the property at a gain of $1250, receiving cash $6000, and a note for the remainder. What was the face of the note?

23. Find the balance of cash on hand on each date as shown by the Cash Book given below.

To find the balance on hand on each date, deduct the amount paid out on Jan. 1 from the balance on hand. To this add the amount received on the 5th, and from this sum subtract the amount paid out on that date, etc., until the 31st. Prove the balance found on the 31st by subtracting the total paid out from the total received.

CASH BOOK.

Receipts.					Payments.
1890.					
Jan. 1.	Balance	800.50	Jan. 1.	For rent	50.00
5.	For Mdse.	10.50	5.	For coal	10.00
10.	Of W. Marsh on a/c	100.00	10.	For labor	15.00
15.	For Mdse.	17.50	15.	To M. G. Troup on a/c	75.00
20.	Of M. L. Barneson a/c	25.00	20.	For horse hire	18.00
25.	For Mdse.	150.50	25.	For postage	3.00
28.	For Mdse.	50.75	28.	For box rent	2.00
30.	For Mdse.	8.25	30.	For laundry bills	5.25
31.	Of E. Parsons on a/c	5.50	31.	For clerk hire	15.00

24. My bank acct. shows the following deposits and amounts checked out. Extend by horizontal addition the total checks drawn on 1st, 10th and 20th. Find the balance in bank at the end of each date and prove final balance same as in example 23.

Chas. E. Dean In a/c First Natl. Bank.

	Deposited.			Drew Out.	
1891.			1891.		
Mar. 1.	Balance	800.00	Mar. 1.	Cks. $35, $100, $1.50, $5, $3.50	***.**
5.	C.	150.50	5.	Ck.	75.50
10.	C.	500.17	10.	Cks. $100.10, $8.90	***.**
15.	C.	25.83	15.	Ck.	85.19
20.	C.	37.18	20.	Cks. $250, $12.50,	***.**
31.	C	19.82	31.	Balance	***.**
		*****.**		Vouchers returned, 11.	***.**
Apr. 1.	Balance in bank	***.**			

MULTIPLICATION.

55. Multiplication is the process of taking one of two numbers as many times as there are units in the other.

One of the numbers is called the ***Multiplicand*** and the other the ***Multiplier***. The numbers are also called ***Factors*** of the ***Product***.

56. The **Multiplicand** is the number multiplied.

57. The **Multiplier** is the number by which the multiplicand is multiplied.

58. The **Product** is the result obtained by multiplying one number by another.

59. The **Sign of Multiplication** is an oblique cross, ×. It is read "*times*" or "*multiplied by,*" and indicates that the numbers between which it is placed are to be multiplied together, or their product obtained. Thus, 5 × 2 is read 5 times 2, or, 5 multiplied by 2.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	2
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	3
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	4
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	5
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	6
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	7
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	8
9	18	27	36	45	54	63	72	8	90	99	108	117	126	135	9
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	10
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	11
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	12
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	13
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	14
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	15

It will be of great advantage to the student to fully master the above table. Any delay caused by following this suggestion will be offset by time gained in subsequent work; such mastery will so increase the rapidity of work in business applications as to greatly lessen the labor of accounting.

60. When either factor is a number within one's thorough knowledge of the multiplication table.

EXAMPLE 1.—Multiply 417 by 5.

OPERATION.

$$\begin{array}{r}417\\5\\\hline 2085\end{array}$$

EXPLANATION.—Write the multiplier 5 below the unit figure of the multiplicand, and multiply each figure of the multiplicand by the multiplier; thus, 5 times 7 = 35, or, 3 tens + 5 ones; write the 5 ones' in the ones' place and reserve the 3 tens to add to the next product; next, 5 times 1 ten are 5 tens, and adding the 3 tens reserved gives 8 tens, which write in ten's place; next, 5 times 4 hundreds are 20 hundreds, or 2 thousands; write a naught, or cipher, in the hundreds' place and the 2 in the thousands' place, thus completing the multiplication and obtaining 2085 as the product of 417 multiplied by 5.

EXAMPLE 2.—Multiply 123 by 12.

OPERATION.

$$\begin{array}{r}123\\12\\\hline 1476\end{array}$$

EXPLANATION.—Multiply each figure of the multiplicand by the multiplier, 12; 12 times 3 = 36, or, 3 tens + 6 ones; write the 6 in ones' place and reserve the 3 tens to add to the next product; next, 12 times 2 tens are 24 tens, and adding the 3 tens reserved gives 27 tens, or 2 hundreds + 7 tens; write the 7 in tens' place and reserve the 2 hundreds to add to the next product; next, 12 times 1 hundred are 12 hundreds, and adding the 2 hundreds reserved gives 14 hundreds, or, 1 thousand 4 hundreds, which write in hundreds' and thousands' places, thus completing the multiplication and obtaining 1476 as the product of 123 multiplied by 12.

Rule.—I. *Write the numbers one below the other, arranged so that figures of like orders stand in the same vertical line.*

II. *Multiply each figure of the upper number, beginning at the right, by the lower number, placing in order the last figure of the product so obtained, and carrying to the next product all figures except the last; continue so doing until the last product is found, which write in full.*

When the multiplier is a number within your thorough knowledge of the multiplication table omit it in the operation, and write only the multiplicand and product, with the usual horizontal line between them. Thus, in the above example, write the multiplicand, then draw a line underneath it and multiply mentally by 12, then write the product in the usual place. This saves time and labor and is more business-like than the other method. Work the following examples by this short method.

EXAMPLES.

61. Find the products of

1. 1325 × 2.
2. 2108 × 11.
3. 511 × 15.
4. 293 × 12.
5. 1801 × 13.
6. 684 × 14.
7. 811 × 16.
8. 1602 × 9.
9. 3006 × 14.
10. 249 × 7.
11. 519 × 8.
12. 1122 × 11.
13. 641 × 13.
14. 7122 × 5.
15. 984 × 8.
16. 2260 × 12.
17. 461 × 14.
18. 3542 × 15.

62. When the multiplier consists of two or more figures.

Multiply 314 by 213.

OPERATION.

$$\begin{array}{rl} 314 & \\ 213 & \\ \hline 942 & = 314 \times 3 = 942 \text{ ones} \\ 314 & = 314 \times 10 = 3140 \text{ ones} \\ 628 & = 314 \times 200 = 62800 \text{ ones} \\ \hline 66882 & = 314 \times 213 = 66882 \text{ ones} \end{array}$$

Explanation.—To multiply 314 by 213 is to multiply 314 by 3 ones, then by 1 ten and then by 2 hundreds. 314 ones × 3 ones = 942 ones; 314 ones × 1 ten = 314 tens or 3140 ones; 314 ones × 2 hundreds = 628 hundreds, or 62800 ones. Adding these partial products together gives 66882, the total product.

It will be observed that in multiplying by the second figure, 1 ten, the first figure of the partial product, 4 tens, is placed directly under it, and that multiplying by the third figure, 2 hundreds, the first figure of the partial product, 8 hundreds, is placed directly under it. This order is observed for the purpose of adding the partial products, as only like orders can be added.

Rule.—I. ***Place the multiplier below the multiplicand, the unit figures in the same vertical line.***

II. ***Beginning with the unit figure, multiply all the figures of the multiplicand by each successive figure of the multiplier, writing the first figure obtained in each partial product directly below the figure by which it was multiplied. Add the partial products.***

EXAMPLES.

63. Find the products of

1. 954 × 317.
2. 629 × 327.
3. 4185 × 514.
4. 1085 × 1006.
5. 999 × 888.
6. 428 × 317.
7. 945 × 218.
8. 754 × 317.
9. 9512 × 654.
10. 545 × 715.
11. 943 × 814.
12. 675 × 854.
13. 788 × 375.
14. 978 × 112.
15. 695 × 718.

16. Find the cost of 315 acres of land at $175 per acre.

17. The average daily sales of a certain business house amounted to $1235. What were the total sales for the year, of 309 business days?

18. The Union Pacific Railway is 1777 miles in length, and was built at an average cost of $106775 per mile. What was the total cost of construction?

19. A certain State whose area is 8315 square miles has an average population of 234 to the square mile. What is the total population of that State?

20. In the year 1889 the average attendance of Indian pupils at the U. S. Government boarding schools was 3581, and at the day schools, 1744. How much did the Government expend in that year for the education and support of these pupils, if the average cost was $175 per pupil for those attending the boarding schools, and $62 per pupil for those attending the day schools?

64. When there are ciphers at the right of either the multiplicand or multiplier, or both.

EXAMPLE.—Multiply 120000 by 7256.

OPERATION.

$$\begin{array}{r} 7256 \\ 120000 \\ \hline 870720000 \end{array}$$

EXPLANATION.—Consider the numbers as reversed in order; thus, 7256 × 120000. Then multiply the 7256 mentally by 12, and to the product, 87072, annex four ciphers, because the 12 was not 12 simple units but 12 ones of the second order of thousands, or tens of thousands.

EXAMPLE 2.—Multiply 2400 by 1200.

OPERATION.

$$\begin{array}{rl} 24 & \!\!|00 \\ 12 & \!\!|00 \\ \hline 48 & \\ 24 & \\ \hline \multicolumn{2}{l}{2880000} \end{array}$$

EXPLANATION.—2400 = 24 × 100; 1200 = 12 × 100. Multiply 24 by 12 which will give 288. Multiply this by 100 and that product by 100 or annex 4 ciphers to the right.

Rule.—*Write the numbers so that the right-hand significant figures of both numbers stand in the same vertical line. Multiply these significant figures together, and to the product annex as many ciphers as there are ciphers at the right of both multiplicand and multiplier.*

EXAMPLES.

65. Find the products of

1. 120 × 40.
2. 1500 × 70.
3. 160 × 130.
4. 1200 × 90.
5. 13000 × 26.
6. 2700 × 420.
7. 900 × 2400.
8. 720 × 300.
9. 1700 × 3152.
10. 4500 × 2400.
11. 33763 × 900.
12. 28401 × 2400.
13. 602300 × 1050.
14. 123202 × 4400.
15. 2317000 × 1809.

66. When the multiplier is 15.

Example.—Multiply 1684 × 15.

OPERATION.
16840
8420
25260

Explanation.—To multiply by 15 is to multiply by one and one-half times 10. Annexing a cipher to the multiplicand multiplies it by 10. Taking one-half of this product, or 8420, is the same as multiplying 1684 by 5. Adding the two partial products gives 25260, the product required.

Rule.—*Annex a cipher to the multiplicand, and to this product add one-half of itself.*

67. When the multiplier consists of two figures and one of the figures is 1, or 1 ten.

Example.—Multiply 1076 × 21.

OPERATION.
1076
2152
22596

Explanation.—Use the multiplicand as the first partial product, or the product of 1076 multiplied by 1, the first figure of the multiplier. Next multiply 1076 by 2, the second figure of the multiplier, and since this is 2 tens, write the first figure of this partial product under the tens or second figure of the other partial product. Adding the two partial products gives 22596, the required product.

Example 2.—Multiply 1352 × 17.

FIRST OPERATION.
13520
9464
22984

SECOND OPERATION.
1352
9464
22984

Explanation.—To multiply 1352 by 17 is to multiply it by 7 and by 10, or by 10 and then by 7. Annexing a cipher to 1352 multiplies it by 10 and gives 13520 as the first partial product. Next multiplying 1352 mentally by 7 gives 9464, the second partial product. Adding the two partial products gives 22984, the product required.

In practice omit the cipher at the right of the multiplicand, which is also the first partial product, and write the first figure of the second partial product one place to the right, as shown in the second operation.

Rule.—*Consider the multiplicand as one of the partial products, and write the other partial product one place to the left of the multiplicand when the units figure of the multiplier is 1, and one place to the right when the tens figure is 1.*

EXAMPLES.

68. Find the products of

1. 1223 × 21.
2. 3786 × 15.
3. 1043 × 19.
4. 4607 × 31.
5. 2042 × 15.
6. 5407 × 61.
7. 2045 × 13.
8. 10054 × 15.
9. 68076 × 51.
10. 20004 × 18.
11. 164763 × 41.
12. 409030 × 15.
13. 847521 × 17.
14. 334325 × 71.
15. 987645 × 14.

BUSINESS EXAMPLES.

(1)

69. Find the cost of
184 # Coffee, 33¢,
510 # Tea, 45¢,
615 # Sugar, 8¢,
184 # Ham, 18¢,
378 # Raisins, 24¢.

(2)

Find the cost of
318 brls. Flour, $6.25,
135 bush. Peaches, $1.75,
850 bush. Quinces, $2.25,
352 brls. Apples, $6.38,
185 bxs. Oranges, $5.25.

(3)

Find the cost of
85 # White Fish, 12¢,
52 # Steak, 18¢,
38 # Roast, 16¢,
75 # Pork Chops, 14¢,
88 # Veal Cutlets, 17¢,
98 # Sausage, 12¢.

(4)

Find the cost of
75 cans Peaches, 16¢,
185 cans Salmon, 18¢,
173 cans Condensed Milk, 18¢,
175 cans Oysters, 55¢,
318 cans Tomatoes, 16¢,
255 cans Cherries, 22c.

(5)

Find the cost of
40 brls. Flour, $6.25,
25 brls. Beef, $14.75,
38 brls. Salt, $1.50,
50 brls. Pork, $9.25,
75 brls. Apples, $6.25,
100 brls. Flour, $5.75.

(6)

Find the cost of
5000 # Lard, 8½¢,
2500 # Sugar, 6½¢,
800 # Japan Tea, 55¢,
625 # Sugar, 7¢,
375 # B. Powder, 45¢,
178 # Coffee, 22¢.

(7)

Find the cost of
32 brls. Beef, $14.25,
500 # Lard, 9¢,
125 # Tea, 55¢,
378 # Coffee, 33¢,
154 brls. Salt, $1.50.

(8)

Find the cost of
75 cans Peaches, 18¢,
75 # White Fish, 12¢,
75 cans Oysters, 55¢,
75 # Sausage, 12¢,
75 brls. Salt, $1.50.

DIVISION.

70. Division is the process of finding how many times one number is contained in another number of the same kind.

One of the numbers is called the *Dividend* and the other the *Divisor*.

71. The **Dividend** is the number divided.

72. The **Divisor** is the number by which the dividend is divided.

73. The **Remainder** is the part remaining when the division is not exact.

74. The **Sign of Division** is the character ÷ ; it indicates that the number placed before it is to be divided by the number after it. Thus, $24 \div 3 = 8$, is read 24 divided by 3 equals 8.

From the above it will be seen that the *dividend* in division corresponds to the *product* in multiplication, and the *divisor* and *quotient* to the *multiplier* and *multiplicand*, or the factors in multiplication.

75. General Law.—*Any change in the dividend produces a like change in the quotient. Any change in the divisor produces an opposite change in the quotient. A like change in both dividend and divisor will not change the quotient.*

Operations in Division are of two kinds, *Short Division* and *Long Division*.

SHORT DIVISION.

76. In **Short Division** the operations are performed mentally and are restricted to those divisions in which the divisor consists of one figure, or is a number coming within one's thorough knowledge of the multiplication table.

77. When the divisor consists of only one figure.

EXAMPLE.—Divide 6482 by 2.

OPERATION.

$$\begin{array}{r} 2\,)\,6482 \\ \hline 3241 \end{array}$$

EXPLANATION.—Write the divisor at the left of the dividend, separating them by a line; next draw a line below the dividend and then divide each figure of the dividend by the divisor, writing the quotient below the figure divided. Thus, 2 is contained in 6 thousands, 3 thousand times; write the 3 below the 6 in thousands' column; next, 2 is contained in 4 hundreds, 2 hundred times; place the 2 below the 4 in hundreds' column; 2 is contained in 8 tens, 4 tens times; write the quotient in tens' column; 2 is contained in 2 ones, 1 time, or once; write one in ones' place, thus completing the division, and obtaining 8241 as a quotient.

78. When the divisor is a number within one's thorough knowledge of the multiplication table.

EXAMPLE.—Divide 31609 by 15.

OPERATION.

$$\begin{array}{r} 15\,)\,\underline{31609} \\ 2107\frac{4}{15} \end{array}$$

EXPLANATION.—Write the terms as before. Divide 31 by 15 and obtain 2, which write below the one as the first figure of the quotient; next, 15 is contained in 16, once; write 1 in hundreds' column; 15 in 10, 0, or no times; write the 0, or cipher, in tens' column; 15 in 109, 7 times, with a remainder of 4 ones, or 4; write the 7 as ones of the quotient, and write the remainder over the divisor in the form of a fraction, and annex the result to the entire part of the quotient, thus completing the division, and obtaining the quotient $2107\frac{4}{15}$.

Rule.—I. *Write the divisor at the left of the dividend with a line separating them.*

II. *Beginning at the left, divide each figure, or each two figures, of the dividend by the divisor, and write the resulting quotient underneath the dividend.*

III. *If after any division there be a remainder, regard this remainder as prefixed to the next figure of the dividend, and divide as before.*

IV. *Should any partial dividend considered, be less than the divisor, place a cipher in the quotient and regard the undivided part as prefixed to the succeeding figure in the dividend and again divide.*

V. *If the division is not exact, write the remainder over the divisor in fractional form, and annex the result to the whole part of the quotient.*

Proof.—*Divide the dividend by the quotient, or multiply the divisor by the quotient. In divisions which are not exact, add the remainder to the product of the divisor and quotient; the sum thus obtained should be the dividend.*

EXAMPLES.

79. Find the quotients of

1. 945 ÷ 3.
2. 1124 ÷ 4.
3. 2646 ÷ 6.
4. 31562 ÷ 8.
5. 2075 ÷ 12.
6. 90090 ÷ 7.
7. 51007 ÷ 11.
8. 16320 ÷ 9.
9. 48674 ÷ 12.
10. 204152 ÷ 11.
11. 58412 ÷ 13.
12. 81052 ÷ 14.
13. 20416 ÷ 15.
14. 84764 ÷ 12.
15. 143258 ÷ 11.

80. When the divisor has ciphers on the right.

EXAMPLE.—Divide 519 by 10.

OPERATION.

$$1|0\;)\;51|9$$

$$51\tfrac{9}{10}$$

EXPLANATION.—Since by the decimal system, numbers increase in value from right to left and decrease from left to right in a tenfold ratio, it follows that to divide a number by 10 it is only necessary to cut off from the right of the number one place. Cutting off one place at the right of 519 gives 51 as a quotient and 9 as a remainder. For like reasons, cutting off two figures divides the number by 100, cutting off three figures divides by 1000, etc. The figures at the left are the quotient, and those on the right, or the ones cut off, are the remainder.

Rule.—*Cut off as many figures at the right of the dividend as there are ciphers in the divisor. The remaining figures will be the quotient, and those cut off, the remainder.*

EXAMPLE 2.—Divide 2684 by 500.

OPERATION.

$$5|00\;)\;26|84$$

$$5\tfrac{184}{500}$$

EXPLANATION.—Cutting off two figures at the right of the dividend and divisor, divides each by 100; this gives a quotient of 26 and a remainder of 84. But since the divisor is not 100 but 500, this quotient is 5 times too large. Dividing 26 hundred by 5 hundred gives 5 as a quotient with a remainder of 100. Adding this partial remainder to the figures cut off gives a true remainder of 184.

Rule.—I. *Cut off the ciphers at the right of the divisor and as many figures from the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor.*

III. *Prefix the remainder, if any, to the figures cut off, and the result will be the true remainder.*

EXAMPLES.

81. Find the quotients of

1. 1164 ÷ 100.
2. 2684 ÷ 30.
3. 9040 ÷ 60.
4. 83251 ÷ 100.
5. 41625 ÷ 500.
6. 94275 ÷ 700.
7. 73024 ÷ 1100.
8. 50642 ÷ 120.
9. 62094 ÷ 400.
10. 12468 ÷ 500.
11. 62500 ÷ 500.
12. 124689 ÷ 900.
13. 620000 ÷ 800.
14. 91000 ÷ 3000.
15. 36000 ÷ 9000.

16. $21735 was received from the sale of a farm at $40 per acre. How many acres did the farm contain?

17. I sell my village home for $3250, my store for $5000, my stock of goods for $11250, receiving in part payment $8775 cash, and for the remainder Iowa prairie land at $30 per acre. How many acres should I receive?

LONG DIVISION.

82. When the divisor is a number larger than can be treated mentally, the following method, called *Long Division*, is employed:

EXAMPLE.—Divide 81439 by 37.

OPERATION.

$$\begin{array}{r l} 37\,)\,81439\,(\,2201\tfrac{2}{37} & \\ 74\phantom{\,(\,2201\tfrac{2}{37}} & \\ \hline 74\phantom{\,(\,2201\tfrac{2}{37}} & \\ 74\phantom{\,(\,2201\tfrac{2}{37}} & \\ \hline 39\phantom{\,(\,2201\tfrac{2}{37}} & \\ 37\phantom{\,(\,2201\tfrac{2}{37}} & \\ \hline 2\phantom{\,(\,2201\tfrac{2}{37}} & \text{Remainder.} \end{array}$$

EXPLANATION.—Write the terms as in short division, and place a line after the dividend to separate it from the quotient, which is now to be written at the right. Then divide the first two figures of the dividend, 81, by the divisor, 37, and obtain 2 as the first figure of the quotient; then subtract from 81 the product of 2 × 37, or 74, obtaining 7 as a remainder; to this remainder annex 4, the succeeding figure of the dividend, which gives 74 as the next partial dividend; the divisor is contained in this dividend twice, or 2 times, giving 2 as the next or second quotient figure; subtracting the product of 2 × 37 from 74, nothing remains; then bring down 3, the next figure of the dividend and as it is less than the divisor, place a 0 in the quotient; next bring down 9, the remaining figure of the dividend which gives 39 as the last partial dividend; the divisor is contained in this dividend once, or 1 time; writing this 1 in the quotient and subtracting the last partial product from the last partial dividend, 2 remains; write this remainder in the form of a fraction at the right of the other figures in the quotient, and $2201\frac{2}{37}$ is the result of dividing 81439 by 37.

Rule.—I. *Write the divisor at the left of the dividend with a curved line between them, and another line at the right of the dividend to separate it from the quotient.*

II. *From the left of the dividend select the least number of figures that will contain the divisor one or more times, and divide. Write the quotient figure thus obtained at the right of the dividend, multiply the divisor by this quotient figure and subtract the product from the partial dividend used. To the remainder annex the succeeding figure of the dividend and divide as before; so continue until the last partial product has been subtracted from the last partial dividend. If there be a remainder place it over the divisor with a line between, and write the resulting fraction as a part of the quotient.*

Proof.—*Multiply the divisor by the quotient, and to the product add the remainder if there be any; the result should equal the dividend.*

EXAMPLES.

83. Find the quotients of

1. 1728 ÷ 48.
2. 2025 ÷ 135.
3. 625 ÷ 125.
4. 1920 ÷ 160.
5. 2268 ÷ 45.
6. 109295 ÷ 28.
7. 52467 ÷ 19.
8. 4762 ÷ 367.
9. 250000 ÷ 793.
10. 87524 ÷ 31.
11. 115680 ÷ 155.
12. 29410 ÷ 251.
13. 666666 ÷ 2154.
14. 93462007 ÷ 1525.
15. 500500500 ÷ 1888.
16. 21416009 ÷ 5407.
17. 11460250 ÷ 999.
18. 87629000 ÷ 11181.

19. If an elephant produces 120 lb. of ivory and the manufactories of Sheffield consume yearly 483000 lb., how many elephants must be killed each year to supply the Sheffield market alone?

20. The log of the yacht Wanderer in circumnavigating the globe in 1880–82, showed 48490 miles run in 280 days, actual running time. What was the average miles run per day?

21. The 2515 miles of canal in the United States cost $170028636. What was the average cost per mile?

22. The total cost of the railroads of the U. S. in 1880 was $5425772550. If the average cost per mile was $62522, how many miles had there been built.

23. In a certain city there are 735 manufacturing concerns. If the total amount of capital invested is $13161870, what is the average capital of each concern?

24. In the year 1883 the United States exported agricultural products to the amount of $550000000. What was the average value per day of these exports?

25. For the year ending September 30, 1887, the exchanges at the Clearing House at New York amounted to $34872848786, and those of the 36 remaining important cities, $17253855702. What was the average of the exchanges per month at the New York Clearing House? What was the average per month at the 36 remaining Clearing Houses?

UNITED STATES MONEY.

84. United States Money is the legal *currency* of the United States, adopted in 1786 and changed by various Acts of Congress since that date; it is sometimes called *Federal Money*.

85. Currency is coin, treasury notes, bank-bills, or any substitute for money, in circulation as a medium of trade.

86. Coin is the standard money of the mints, its value being established by law.

87. The Dollar is the *unit* of United States money. Dollars are written as integers, with the sign ($) prefixed; the lower denominations are written as *decimals*, dimes being *tenths*, cents *hundredths*, and mills *thousandths* of a dollar. Thus, 15 dollars, 1 dime, 5 cents, 5 mills, is written $15.155.

In business records and papers, *cents* are often written as fractions of a dollar; the *half-cent* is expressed either as a fraction (½), or as 5 mills. Thus, $15.75 may be written 15\frac{75}{100}$; 12½ cents, $.12½, or $.125.

88. The denominations and scale of United States money are shown in the following

TABLE.

10 mills = 1 cent (c. or ct.).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 eagle (E.)

The *Dime* is a coin, but its name is never used in reading United States money. The Mill is not coined; it is used only as the decimal of the cent, which is the smallest money of the mint and the smallest recognized in business.

UNITED STATES COINS.

89. The **Coins** of the United States, authorized by various Acts of Congress, are of *gold*, *silver*, *copper-nickel*, and *bronze*.

90. The **Gold Coins** of the United States are as follows:

1. The *Double Eagle;* value, $20; weight, 516 Troy grains.

2. The *Eagle;* value, $10; weight, 258 Troy grains.

3. The *Half-Eagle;* value, $5; weight, 129 Troy grains.

4. The *Three Dollar* piece; value, $3; weight, 77.4 Troy grains.

5. The *Quarter-Eagle;* value, $2.50; weight, 64.5 Troy grains.

6. The *One Dollar* piece; value, $1; weight, 25.8 Troy grains.

All United States gold coins are made of $\frac{9}{10}$ pure gold, and $\frac{1}{10}$ alloy of copper and silver, the alloy being used to toughen the metal, so as to reduce the loss from abrasion. The alloy used is never more than $\frac{1}{10}$ part silver.

United States gold coins of standard weight are legal tender for all debts.

91. The **Silver Coins** of the United States are as follows:

1. The *Dollar;* value, $1.00; weight, 412.5 Troy grains.

2. The *Half-Dollar;* value, 50¢; weight. 192.9 Troy grains.

3. The *Quarter-Dollar;* value, 25¢; weight, 96.45 Troy grains.

4. The *Dime;* value, 10¢; weight, 38.58 Troy grains.

The value of gold and silver coins is based mainly on their weight and fineness, or the amount of *pure* metal used. Silver coins are made of $\frac{9}{10}$ pure silver and $\frac{1}{10}$ alloy of copper.

United States silver dollars are *legal tender* for all sums not otherwise provided for by contract. The smaller silver coins are *legal tender* for all sums not exceeding ten dollars.

92. The **Copper-Nickel Coins** of the United States are as follows:

1. The *Five-Cent* piece, called the *nickel;* weight, 77.16 Troy grains.

2. The *Three-Cent* piece; weight, 30 Troy grains.

The 5¢ and 3¢ coins are composed of $\frac{3}{4}$ copper and $\frac{1}{4}$ Nickel.

93. The **Bronze Coin.**—The only bronze coin now issued from the mint is the one cent piece, weighing 48 Troy grains, and composed of $\frac{95}{100}$ copper and $\frac{5}{100}$ tin and zinc.

The 5¢ and 3¢ nickel coins, and the 1¢ bronze coin, are called minor coins; and while they are legal tender for all sums not exceeding twenty-five cents, their value is not a bullion value, as in case of coins of gold and silver, but an arbitrary value fixed for commercial convenience.

UNITED STATES PAPER MONEY.

94. The **Paper Money** of the United States consists of *Treasury Notes, Treasury Certificates,* and *National Bank Bills.*

95. **Silver Certificates.**—Any holder of silver dollars, to the amount of ten dollars or more, may deposit the same with the Treasurer or Assistant Treasurer of the United States and obtain therefor *Silver Certificates,* which are receivable for duties, taxes, and all public debts; and any holder of the smaller silver coins to the amount of twenty dollars, or any multiple thereof, may obtain therefor lawful money at the office of the Treasurer or of any Assistant Treasurer.

96. United States Treasury Notes.—Treasury Notes, or Greenbacks, are in the same denominations as the Bills of National Banks, with the addition of those of $5000 and $10000 value respectively. They are *legal tender* for all debts, except customs or duties, and interest on the public debt, and are usually receivable for these also, being convertible into coin on demand when presented in sums of fifty dollars or more.

97. National Bank Bills.—National Bank Bills are the notes issued by National Banks, under the supervision of Government, and these bills are in denominations of $1, $2, $5, $10, $20, $50, $100, $500, and $1000, and being secured by deposits of Government Bonds with the United States Treasurer, and redeemable on demand with lawful money, are usually received for all dues, but yet *are not legal tender;* and a debt cannot be paid with these notes if the creditor states as his reason for their rejection that they are not lawful money.

REDUCTION OF UNITED STATES MONEY.

98. To reduce dollars to cents.

EXAMPLE.—Reduce 5 dollars to cents.

EXPLANATION.—Since there are 100 cents in 1 dollar, in 5 dollars there are 5 times 100 cents, or 500 cents.

Rule.—*Add two ciphers to the dollars.*

99. To reduce cents to dollars.

EXAMPLE.—Reduce 1500 cents to dollars.

EXPLANATION.—Since 100 cents make 1 dollar, there are as many dollars in 1500 cents as 100 cents are contained times in 1500 cents, or 15 times, equal to 15 dollars.

Rule.—*Divide the cents by 100, by pointing off two places from the right.*

EXAMPLES.

100. Reduce

1. 6 dollars to cents.
2. 111 dollars to cents.
3. 241 cents to dollars.
4. 1044 cents to dollars.
5. 21468 cents to dollars.
6. 1800 cents to dollars.
7. 51000 cents to dollars.
8. 9876 cents to dollars.
9. $100.98 to cents.
10. $3.75 to cents.
11. $26.53 to cents.
12. $157.32 to cents.

ADDITION AND SUBTRACTION.

101. To add or subtract United States money.

Rule.—*Write dollars under dollars and cents under cents; then add or subtract as in simple numbers.*

EXAMPLES.

102. *1.* Add ten dollars twenty cents, six dollars forty-eight cents, fourteen dollars twenty-six cents, eleven dollars eighty cents, and forty-six dollars ten cents.

2. Subtract seven hundred sixty-five dollars nineteen cents, from nine hundred ten dollars eight cents.

3. A farmer sold produce as follows: wheat, for $761.25; oats, $382.40; barley, $816.09; buckwheat, $186.92; corn, $1127.50; potatoes, $663.11; hay, $400.50. What were his entire sales?

4. A lady bought groceries to the amount of $6.85; meats, $2.11; dry goods, $31.15; carpets, $167.25; millinery, $13.57. What was the total amount of her purchases?

5. A student expends for tuition and supplies, $118.75; for board, $167.50; for clothes, $57.25; for entertainment and church, $28.42; for charity, $6.15. What amount does he expend?

6. The expenses of my house are as follows: for interest, $167.50; taxes, $103.29; repairs, $56.82; insurance, $11.35; water rent, $11.25; and gas, $27.08. What are my total expenses?

Under some circumstances it is desirable to write United States money, expressed in dollars and cents, without the $ sign and the decimal point, with the decimal part placed slightly above that expressing the integers or dollars; as $5.25 may be written 5^{25}; thirteen dollars and eight cents may be written 13^{08}. This is advisable only where the sum of several items is to be found by horizontal addition.

The amount in each of the following examples is to be found by horizontal addition.

7. Add 15^{16}, 2^{97}, 11^{46}, 107^{90}, 9^{28}, 81^{45}, 123^{92}, 6^{01}, 15^{30}, and 11^{11}.

8. Add 346^{50}, 291^{75}, 100^{31}, 269^{11}, and 80^{93}.

9. What is the sum of 2165^{84}, 72^{43}, 90^{20}, 117^{65}, 500^{60}, and 1127^{14}?

10. My bills for a year are: for groceries, 283^{51}; meats, 135^{11}; miller's products, 76^{75}; coal, 41^{20}; kindling, 4^{50}; milk, 47^{25}; servant, 217; incidentals, 91^{54}. What are my expenses?

MULTIPLICATION.

103. To multiply United States money.

EXAMPLE.—At $2.25 a yard, what will 8 yards of cloth cost?

OPERATION.

```
$2.25
    8
-----
$18.00
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EXPLANATION.—The multiplicand $2.25 is equal to 225 cents. 8 times 225 cents equals 1800 cents, or $18.00.

Rule.—*Multiply as in simple numbers, and from the right of the product point off as many places as there are places at the right of the decimal points in both numbers.*

Money is a concrete expression; therefore in critical analysis of its multiplication, the money *cost* or *price* of an article is a concrete multiplicand, the number of things bought or sold is an abstract multiplier, and their product is concrete and of the denomination of the multiplicand. But since the money scale is *decimal*, these terms may be interchanged for convenience.

EXAMPLES.

104. *1.* What will be the amount of the following purchases: 147 cd. hard wood, at $5.75 per cd.; 206 cd. soft wood, at $4.25 per cd.; 4 car loads slab wood, each containing 16 cd., at $2.75 per cd.; 816 tons hard coal, at $5.15 per ton; and 536 tons soft coal, at $3.85 per ton?

2. Bought 215 brl. superfine flour, at $4.85 per brl.; 355 brl. extra flour, at $5.15 per brl.; 132 brl. rye flour, at $4.90 per brl.; 210 brl. corn meal, at $3.70 per brl.; and 642 sacks graham flour, at 88¢ per sack. What was the total cost?

3. An invoice of six pieces of gingham of 51, 49, 50, 54, 49, and 51 yd. respectively, was sold at $.09 per yd. What was the amount of the sale?

4. Six men worked 19 days each, at $1.90 per day; 24 days each, at $1.80; 11 days each, at $1.55; and 31 days each, at $1.15. How much was earned by all in the entire time?

5. A laborer received $184.55 as a balance due him for his season's work. He paid a debt of $19.25; bought 8 yd. cloth, at $1.25 per yd.; 2 suits of clothes, at $13.25 per suit; hosiery and gloves for $2.85; 4 tons coal, at $5.65 per ton; 2 cd. wood, at $3.80 per cd.; 3 brl. flour, at $4.75 per brl.; 628 pounds of pork, at 6¢ per lb.; and loaned the remainder of his money. How much did he loan?

DIVISION.

105. To divide United States money.

EXAMPLE.—1. If $11421.75 be divided equally among five persons, what will be the share of each?

OPERATION.

5) $11421.75
$2284.35

EXPLANATION.—Ignoring the decimal point in the dividend, which is the same as reducing the dividend to cents, and dividing we obtain 228437 cents, or 2284 dollars and 37 cents.

EXAMPLE 2.—A grocer paid $18.72 for a barrel of sugar. If the sugar cost $.06 a pound, how many pounds did the barrel contain?

OPERATION.

$.06) $18.72
312

EXPLANATION.—If 1 pound costs 6 cents, $18.72 or 1872 cents will buy as many pounds as 6 is contained times in 1872, or 312 times or 312 pounds.

Rule.—*Divide as in simple numbers, and point off from the right of the quotient as many places as the number of places at the right of the point in the dividend exceed those at the right of the point in the divisor.*

In case exact quotients are not obtained in division of dollars, add two decimal ciphers and continue the quotient to cents; if not then exact add one cent if the mills be 5 or more, but if less than 5, reject the mills.

EXAMPLES.

106. Find the quotients of

1. $1325 ÷ 5.
2. $786.04 ÷ $.44.
3. $4765 ÷ $1.25.
4. $8032.50 ÷ 75.
5. $1203 ÷ $.35.
6. $542.50 ÷ 35.
7. $1242.50 ÷ $8.75.
8. $10000 ÷ $.55.
9. $2052.35 ÷ 13.
10. $9137 ÷ $2.25.
11. $4859.76 ÷ 125.
12. $21168 ÷ $1.35.
13. $2284.35 ÷ 113.
14. $315.93 ÷ $2.15.
15. $65877 ÷ $1.125.
16. $765856.35 ÷ 365.

17. B sold 195 acres of land at $105.25 per acre, and divided the proceeds equally among thirteen persons. What sum did each receive?

18. A dealer bought wheat at $.95 per bu., oats at $.45 per bu., and corn at $.65 per bu. He paid $332.50 for the wheat, $191.25 for the oats, and $113.75 for the corn. How many bushels did he buy in all?

TO CALCULATE INTEREST.

U. S. MONEY METHOD.

107. Most teachers in business schools and commercial departments are constantly confronted with the fact that students who are studying book-keeping should be taught to calculate interest long before the subject of interest is reached in the regular way in the Arithmetic.

In recognition of this a part of the subject of interest is here introduced in connection with the fundamental rules and based on the United States money method of calculating interest. No reference is made to decimal fractions nor to the principles of percentage, but after showing that one cent is the interest or use of one dollar for 60 da., the subject is presented by tables of aliquots, so arranged that the student is led to fully appreciate the subject, and as a result will in a short time be able to calculate interest with accuracy and rapidity.

108. **Interest** is the price paid for the use of money.

109. The **Principal** is the money for the use of which interest is paid.

110. The **Rate of Interest** is the annual rate per cent.

Rate per cent. per annum means that a certain number of *cents* is paid for the use of *one dollar* for *one year.*

111. **Bank Discount** is the simple interest on negotiable paper, paid in advance.

All problems in bank discount are worked by the rules governing interest, as the same principles involved in calculations of the latter are applicable to the former.

112. Money loaned at six per cent. per annum (written 6%) means that one dollar loaned for one year will produce six cents interest for its use. As $1.00 at 6% will produce 6 cents interest in one year (written .06—see United States Money, page 30), it therefore follows that $1.00 for one month at 6% will produce one-half cent or five mills (written .005) and for 2 months, $1.00 will produce 1 cent interest (written .01).

Using two months or 60 days as a basis or standard of comparison, it is evident that, as the interest on $1.00 for 60 days is .01, for 2 dollars the interest will be *two times* 1 cent, or 2 cents (written .02), on 5 dollars, five times one cent or 5 cents (.05), and on 525 dollars, *five hundred and twenty-five times* 1 cent or 525 cents ($5.25), etc.

General Rule.—I. *The interest on any amount of money at 6% for 60 days is as many times one cent as the number of dollars is times one dollar.*

II. *The interest on any amount of money for any number of days equals such a part of the interest for 60 days as the number of days is a part of 60 days.*

113. To calculate interest at 6% on a 360 day basis.

ALIQUOT PARTS OF 60 DAYS.

The aliquot parts of a number are the even parts of that number.

The interest on any principal, or the sum upon which interest is computed, at 6% for $\left\{\begin{array}{l} 30 \text{ days} = \frac{1}{2} \\ 20 \text{ days} = \frac{1}{3} \\ 15 \text{ days} = \frac{1}{4} \\ 12 \text{ days} = \frac{1}{5} \\ 10 \text{ days} = \frac{1}{6} \\ 6 \text{ days} = \frac{1}{10} \end{array}\right\}$ of the interest for 60 days.

Rule.—*Find the interest on the given principal for 60 days by pointing off two places from the right of dollars; then take such a part of the interest for 60 days as the number of days is a part of 60 days.*

ALIQUOT PARTS OF 6 DAYS.

The interest on any principal for $\left\{\begin{array}{l} 3 \text{ days} = \frac{1}{2} \\ 2 \text{ days} = \frac{1}{3} \\ 1 \text{ day} = \frac{1}{6} \end{array}\right\}$ of the interest for 6 days.

Rule.—*Find the interest on the given principal for 6 days by pointing off three places from the right of dollars; then take such a part of the interest for 6 days as the numbers of days is a part of 6 days.*

EXAMPLE 1.—Find the interest on $813 for 60 days.

OPERATION.

Int. on $1.00 for 60 ds. = 1¢
Int. on $813 = 813 × 1¢ = 813¢
813¢ ÷ 100¢ = $8.13.
Or,
Int. on $813 = $8|13.

EXPLANATION.—Since the interest on $1.00 for 60 ds. is 1¢ (written .01) on $813 the interest is 813 × 1¢ or 813¢, which equals $8.13, found by pointing off two places for cents.

The direct way, however, is to cut off two places from the right, thereby changing cents to dollars and cents.

EXAMPLE 2.—Find the interest on $1200 for 57 days.

OPERATION.

Int. for 60 ds. =	$12	00
30 ds. = $\frac{1}{2}$ =	6	00
15 ds. = $\frac{1}{4}$ =	3	00
12 ds. = $\frac{1}{5}$ =	2	40
57 ds. =	$11	40

EXPLANATION.—The interest on $1200 for 60 days is $12. The highest aliquot parts of 60 found in 57 days are 30 days, 15 days, and 12 days. Hence, $\frac{1}{2}$ plus $\frac{1}{4}$ plus $\frac{1}{5}$ of the interest for 60 days, or $11.40 is the interest on $1200 for 57 days.

Observe that in the operation the work is proved by adding 30, 15 and 12 days, respectively, which sum equals 57 days.

To find the highest aliquot parts in any number, first deduct from the number the highest aliquot part in that number, and then from the remainder deduct the highest aliquot part in that number, and so on to the end.

EXAMPLE.—Give the aliquot parts of 119 days.

EXPLANATION.—By the table it will be seen that, after deducting 60 days from 119 days that 30 days is the highest aliquot in the remainder of 59 days. Deducting 30 days, next observe that 20 is the next highest in the remaining 29 days, then 6 days, then 3 days.

Students who are not familiar with fractions should appreciate that to get $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ or $\frac{1}{10}$ of a number is to divide the number by 2, 3, 4, 5, 6, or 10, as the case may be. Thus, to get $\frac{1}{6}$ of 180: $180 \div 6 = 30$.

EXAMPLE 3.—Find the interest on $1800 for 5 days.

OPERATION.

Int. for 6 days =	$1	800
3 ds. = $\frac{1}{2}$ =		90
2 ds. = $\frac{1}{3}$ =		60
Int. for 5 ds. =	$1	50

EXPLANATION.—The interest for 6 days is found by pointing off three places from the right of $1800, which gives $1.800. The aliquots for 3 days and 2 days are $\frac{1}{2}$ and $\frac{1}{3}$; and $\frac{1}{2}$ and $\frac{1}{3}$ of $1.80 are 90¢ and 60¢, which make a total of $1.50.

EXAMPLE 4.—Find the interest on $3600 for 79 days.

OPERATION.

Int. for 60 ds. =	$36	00
15 ds. = $\frac{1}{4}$	9	00
3 ds. = $\frac{1}{2}$ of 6 ds.	1	80
1 day = $\frac{1}{6}$ of 6		60
Int. for 79 ds. =	$47	40

EXPLANATION.—The interest on $3600 for 60 days equals $36.00. To this add $\frac{1}{4}$ of $36.00 for 15 days, which is $9.00. For 3 days and 1 day take respectively $\frac{1}{2}$ and $\frac{1}{6}$ the interest for 6 days, which amounts are $1.80 and 60¢. The sum of $36., $9., $1.80 and 60¢ is $47.40, and equals the interest for 79 days.

EXAMPLES.

114. Find the interest at 6%

(*1.*)
On $900 for 60 days.
On $900 for 30 days.
On $900 for 20 days.
On $900 for 15 days.
On $900 for 12 days.
On $900 for 10 days.
On $900 for 6 days.
Find the total interest.

Find the interest at 6%

(*2.*)
On $1500 for 60 days.
On $1500 for 6 days.
On $1500 for 3 days.
On $1500 for 2 days.
On $1500 for 1 day.
On $1500 for 72 days.
On $1500 for 7 days.
Find the total interest.

3. On $450 for 30 days.
4. On $624 for 15 days.
5. On $750 for 12 days.
6. On $150 for 10 days.
7. On $960 for 6 days.
8. On $2400 for 3 days.
9. On $3000 for 2 days.
10. On $72000 for 1 day.

11. On $2400 for 66 days.
12. On $1800 for 33 days.
13. On $750 for 67 days.
14. On $750 for 77 days.
15. On $960 for 53 days.
16. On $1260 for 47 days.
17. On $3300 for 21 days.
18. On $8400 for 100 days.

115. When the number of days is an even number of times 60

$$\text{The interest on any principal at 6\% for} \left\{ \begin{array}{l} 180 \text{ days} = 3 \\ 120 \text{ days} = 2 \end{array} \right\} \text{times the interest for 60 days.}$$

EXAMPLE.—Find the interest on $750 for 120 days.

OPERATION.

Int. for 60 days = $7|50
2
2 times $7.50 = $15|00

EXPLANATION.—The interest on $750 for 60 days = $7.50. Since 120 days = 2 times 60 days, the interest for 120 days will be 2 × $7.50, or $15.

Rule.—*Find the interest for 60 days; then take the interest for 60 days as many times as the number of days is times 60.*

116. When the number of days is an even number of times 6.

$$\text{The interest on any principal at 6\% for} \left\{ \begin{array}{l} 12 \text{ days} = 2 \\ 18 \text{ days} = 3 \\ 24 \text{ days} = 4 \\ 30 \text{ days} = 5 \\ 36 \text{ days} = 6 \\ 42 \text{ days} = 7 \\ 48 \text{ days} = 8 \\ 54 \text{ days} = 9 \\ 66 \text{ days} = 11 \\ 72 \text{ days} = 12 \end{array} \right\} \text{times the interest for 6 days.}$$

Example.—Find the interest on $750 for 42 days.

OPERATION.

Int. for 6 days =		.750
		7
7 times 75¢ =	$5	25

Explanation.—The interest on $750 for 6 days = 75¢. Since 42 days = 7 times 6 days, the interest for 42 days will be 7 times 75¢, or $5.25.

Rule.—*Find the interest for 6 days; then take the interest for 6 days as many times as the number of days is times 6.*

EXAMPLES.

117. Find the interest at 6% on

1. $925 for 54 days.
2. $340 for 36 days.
3. $420 for 48 days.
4. $8000 for 180 days.
5. $3546 for 18 days.
6. $375 for 66 days.
7. $1500 for 96 days.
8. $960 for 24 days.
9. $3000 for 240 days.
10. $4500 for 360 days.

118. When the number of days is an aliquot part more or less than 60.

The interest on any principal at 6 % for	54 days = $\frac{1}{10}$	*Less* than the interest on the same amount for 60 days.
	50 days = $\frac{1}{6}$	
	48 days = $\frac{1}{5}$	
	45 days = $\frac{1}{4}$	
	40 days = $\frac{1}{3}$	
	30 days = $\frac{1}{2}$	
	66 days = $\frac{1}{10}$	*More* than the interest on the same amount for 60 days.
	70 days = $\frac{1}{6}$	
	72 days = $\frac{1}{5}$	
	75 days = $\frac{1}{4}$	
	80 days = $\frac{1}{3}$	
	90 days = $\frac{1}{2}$	

Example.—Find the interest on $3670 for 48 days.

OPERATION.

Interest for 60 ds. =	$36	70
Interest for 12 ds. = $\frac{1}{5}$	7	34
Interest for 48 ds. =	$29	36

Explanation.—The interest for 60 ds. is $36.70. The difference between 60 ds. and 48 ds. is 12 ds. The interest for 12 ds. is $\frac{1}{5}$ of the interest for 60 ds., or $7.34; $36.70—$7.34 = $29.36.

Rule.—*Find the interest for 60 days; then add to or take from the interest for 60 days such a part of it as the difference between the given number of days and 60 days is a part of 60 days.*

119. When the number of days is an aliquot part, more or less than 6.

The interest on any amount at 6 % for

5 days = $\frac{1}{6}$
4 days = $\frac{1}{3}$
3 days = $\frac{1}{2}$

Less than the interest on the same amount for 6 days.

7 days = $\frac{1}{6}$
8 days = $\frac{1}{3}$
9 days = $\frac{1}{2}$

More than the interest on the same amount for 6 days.

Example.—Find the interest on $360 for 5 days.

OPERATION.

Int. for 6 days =	$0	360
Int. for 1 day = $\frac{1}{6}$ =		06
Int. for 5 days =	$0	30

Explanation.—Since the difference in days (1 day) is $\frac{1}{6}$ of 6 days, subtract $\frac{1}{6}$ of 36¢, the interest for 6 days, from itself which gives (36¢ — 6¢) 30¢.

Rule.—*Find the interest for 6 days; then, add to or take from the interest for 6 days such a part of it as the difference between the given number of days and 6 days is a part of 6 days.*

EXAMPLES.

120. Find the interest at 6 % on

1. $850 for 120 days.
2. $240 for 5 days.
3. $375 for 78 days.
4. $350 for 3 days.
5. $252 for 66 days.
6. $4555 for 180 days.
7. $3600 for 4 days.
8. $1750 for 48 days.
9. $1500 for 54 days.
10. $375 for 36 days.

121. When the principal is expressed in dollars and cents.

Example.—Find the interest on $12.24 for 75 days.

OPERATION.

Int. for 60 days =	$	1224
Int. for 15 days = $\frac{1}{4}$ =		0306
Int. for 75 days =	$	1530

Explanation. — Proceed according to rule under Art. 118. Observe that dollars and cents, in the notation of United States money, decreases from left to right in a ten fold ratio. The interest on $12.24 for 60 days is found by pointing off two places from the right of *dollars* which gives $.1224 (12 cents, 2 mills and 4 parts of mills, each part being one-tenth of a mill). The interest for the 15 ds. is 3¢ and 6 parts of a mill. Adding like in simple numbers gives 15¢ and 3 mills, or, in business, 15¢. If the mills are 5 or more, another cent is added. If less than 5 mills are computed, the odd mills are dropped.

Rule.—I. *Cut off from the right of dollars (indicated by the separatrix dividing dollars from cents) two places for 60 days, and three places for 6 days. The result will be the interest expressed in dollars at the left of the vertical line, and in cents, mills and parts of mills at the right of the line.*

II.—*Take such a part of the interest in dollars, cents, mills and parts of mills for 60 days, or 6 days, thus obtained as the number of days given is an aliquot part of 60 or of 6 days.*

Carry out divisions to parts of mills. If the sum is 5 mills or more add 1 cent to cents. If less than 5 mills, discard the excess below cents.

EXAMPLES.

122. Find the interest on

1. $125.15 for 60 days.
2. $503.05 for 66 days.
3. $1275.25 for 6 days.
4. $31319.40 for 33 days.
5. $4596.83 for 18 days.
6. $7.50 for 9 days.
7. $12.13 for 15 days.
8. 135.48 for 27 days.
9. $1675.42 for 93 days.
10. $21365.18 for 186 days.

123. To find the interest at any rate per cent.

Example.—Find the interest on $2400 for 66 days at 7%.

OPERATION.

$24	00 = int. 60 days at 6%
2	40 = int. 6 days at 6%
26	40 = int. 66 days at 6%
4	40 = $\frac{1}{6}$ of $26.40 = 1%
$30	80 = int. for 66 d. at 7%

Explanation.—Find the interest on $2400 at 6% for 66 days as heretofore explained, which gives $26.40. As 7% is 1% more than 6%, it is evident that the interest required is $\frac{1}{6}$ of $26.40. $26.40 ÷ 6 = $4.40, the interest at 1%, which added to $26.40 = $30.80 the interest at 7%.

Rule.—*Find the interest at 6%; then add to or subtract from the amount of interest thus obtained such a part of itself as the difference in the rate per cent. is a part of 6%.*

124. The following table will serve as a guide to the different rates:

For 7%—Excess over 6% being 1% *add* $\frac{1}{6}$ to int. for 6%.
For 8%—Excess over 6% being 2% *add* $\frac{1}{3}$ to int. for 6%.
For 9%—Excess over 6% being 3% *add* $\frac{1}{2}$ to int. for 6%.
For 5%—Shortage of 6% being 1% *subtract* $\frac{1}{6}$ from int. for 6%.
For 10%—Excess over 6% being 4% *add* $\frac{2}{3}$ to int. for 6%.

For 10%—Excess being $\frac{2}{3}$ find $\frac{1}{3}$ and multiply it by 2; or find $\frac{1}{6}$ of the interest at 6% and then remove the decimal point in the quotient one place to the right.

EXAMPLES.

125. Find the interest on

1. $695, for 69 days, at 7%.
2. $546, for 72 days, at 8%.
3. $1382.50, for 96 days, at 9%.
4. $500, for 15 days, at 5%.
5. 101.80, for 60 days, at 10%.
6. $99, for 33 days, at 5%.
7. $780, for 120 days at 10%.
8. $1350, for 150 days, at 8%.
9. $775, for 180 days, at 7%.
10. $938.20, for 10 days, at 9%.

MISCELLANEOUS EXAMPLES.

126. Find the interest at 6% on

1. $1750, for 15 days.
2. $1125, for 24 days.
3. $742.50, for 30 days.
4. $900, for 93 days.
5. $660, for 63 days.
6. $136.42, for 33 days.
7. $1000, for 21 days.
8. $2000, for 12 days.
9. $351.23, for 40 days.
10. $1368, for 50 days.

Find the discount at 6% on

11. $93.40, for 150 days.
12. $550, for 75 days.
13. $842.50, for 45 days.
14. $800, for 27 days.
15. $1725, for 57 days.
16. $125, for 55 days.
17. $3741.85, for 6 days.
18. $5178, for 9 days.
19. $732, for 11 days.
20. $1174.51, for 42 days.
21. $3750.18 for 13 days.

22. Elmer Bliss owes Darwin & Snyder on account $600, at 6%, which is past due 87 days. How much does he owe for both account and interest?

23. Lee, Dood & Co. borrow on their note $500, at 6% for 36 days without grace. Find amount of interest at maturity.

24. Victor Bell paid interest at 6% for 123 days on a mortgage of $750.98, in cash. Find the amount of cash paid out.

25. Discounted note of Perry C. Gains, indorsed by Wm. L. Cooper, for $938.12, and received cash for the face of note less bank discount at 6% for 49 days. How much is the discount and how much cash did I receive?

FACTORS.

127. The **Factors** of a number are such numbers as multiplied together will produce that number; thus, 3 and 4, also 3, 2, and 2 are factors of 12.

128. A **Composite Number** is one composed of factors; thus, 30 is composed of the factors 5, 2, and 3.

129. A **Prime Number** is one not composed of factors; thus, 3, 5, 7, 11, and 13, are prime numbers, because they cannot be resolved or separated into two or more factors.

A prime number has no divisor except itself and unity. 2 is the only even number that is prime.

130. A **Prime Factor** is a *prime* number used as a factor.

The prime factors of a number are also exact divisors of that number.

Any number is divisible by itself and unity.

Any even number is divisible by 2.

Any number ending with 0 or 5 is divisible by 5.

Any number ending in 0, 00, 000 etc. is divisible by 10, 100, 1000 etc., respectively.

Any number is divisible by 4 when the two right-hand digits are divisible by 4.

Any number is divisible by 3 if the sum of its digits is divisible by 3.

131. To aid the pupil in determining the prime factors of a *composite* number the following table is given:

TABLE OF PRIME NUMBERS FROM 1 TO 211.

1	13	37	61	89	113	151	181
2	17	41	67	97	127	157	191
3	19	43	71	101	131	163	193
5	23	47	73	103	137	167	197
7	29	53	79	107	139	173	199
11	31	59	83	109	149	179	211

132. To find the prime factors of a composite number.

EXAMPLE.—Find the prime factors of 4290.

OPERATION.

5) 4290
2) 858
3) 429
11) 143
13

EXPLANATION.—Observe that the given number ends with a cipher, hence it is exactly divisible by the prime number 5, by which divide it; next, observe that the quotient ends with an even number, and is, therefore, exactly divisible by 2, so divide by 2; then observe that 3 will exactly divide the quotient 429; divide by it, obtaining 143, which divide by 11, obtaining 13, which is a prime factor of the given number. All the divisors and the last quotient being prime numbers, they together constitute the prime factors of 4290.

Rule.—*Divide by any prime number that is exactly contained in the dividend; divide the resulting quotient in the same manner, and continue this until the final quotient is a prime number. The prime divisors and the last quotient will be all the prime factors of the dividend.*

EXAMPLES.

133. Resolve the following numbers into their prime factors.

1. 27.
2. 117.
3. 165.
4. 93.
5. 2376.
6. 108.
7. 172.
8. 180.
9. 360.
10. 512.
11. 1050.
12. 144.
13. 15625.
14. 22464.
15. 181790.

DIVISORS.

134. An **Exact Divisor** of a number is one that will divide the number without a remainder; thus, 3 is an exact divisor of 30, 15, 12; 5 of 15, 25 and 40.

135. A **Common Divisor** of two or more nnmbers is one that will exactly divide each of them without a remainder; thus, 3 is a common divisor of 6, 9, 12 and 15.

136. The **Greatest Common Divisor** of two or more numbers is the greatest number that will divide each of the numbers without a remainder; thus, 6 is the greatest common divisor of 12, 30 and 36.

137. To find the greatest common divisor.

EXAMPLE.—Find the greatest common divisor of 12 and 18.

OPERATION.

$12 = 2 \times 3 \times 2$

$18 = 2 \times 3 \times 3$

$2 \times 3 = \text{g. c. d.}$

EXPLANATION.—As the greatest common divisor is composed of all the prime factors common to both 12 and 18, it will be seen by resolving the numbers into their prime factors that 2 and 3 are common to both; the greatest common divisor is composed of the factors 2 and 3 which give 6 as a product.

When the greatest common divisor cannot be determined by inspection, the operation indicated below should be followed, which depends upon the following:

138. Principles.—*1. A divisor of a number is a divisor of any number of times (multiple) that number.*

Thus, 3 is a divisor of 9 and also of *two times* 9, *three times* 9 etc., as 18, 27, and 36.

2. A common divisor of two or more numbers is a divisor of their sum and also of their difference.

Thus, 3 is a divisor of 9 and of 18. It will divide their sum (27 ÷ 3 = 9) and also their difference (9 ÷ 3 = 3).

3. The greatest common divisor of two numbers is equal to the greatest common divisor of the smaller number and the remainder after division.

Thus, 4 is the g. c. d. of 12 and 20, and it is also the g. c. d. of 12 and of the remainder after dividing 20 by 12 (20 ÷ 12 = 1 + rem. 8), and the g. c. d. of 8 and 12 is 4.

EXAMPLE.—Find the greatest common divisor of 16 and 44.

OPERATION.

```
16 ) 44 ( 2
     32
     --
     12 ) 16 ( 1
          12
          --
           4 ) 12 ( 3
               12
               --
```

EXPLANATION.—If 16 will divide 44 without a remainder it will be the greatest common divisor, since no number can have a divisor greater than itself. By trial 16 is contained twice in 44 with a remainder 12, hence it is not the g. c. d. Since the g. c. d. of 16 and 44 is a common divisor of 16 and 44 it is also a divisor of 32 (Princp. 1) and of the difference 12 (Princp. 2). If 12 will divide 16 it will also divide 32 (2 times 16) and 44 (32 + 12) and will be the greatest common divisor sought. But 12 will divide 16 once with a remainder 4, hence 12 is not the g. c. d. By Princp. 3 the g. c. d. of 16 and 44 is the g. c. d. of 12 and 16 and also of 4 and 12 and cannot be greater than 4. Now the g. c. d. of 4 and 12 is 4 and therefore 4 is the g. c. d. of 16 and 44.

Rule.—*Divide the greater number by the less, and the last divisor by the last remainder, and so continue to divide until a quotient is obtained without a remainder. The last divisor obtained will be the greatest common divisor.*

Where more than two numbers are given, first find the g. c. d. of any two of them; then of the g. c. d. thus found and the remaining number.

EXAMPLES.

139. Find the g. c. d. of

1. 57 and 95.
2. 51 and 85.
3. 65 and 143.
4. 125 and 625.
5. 144 and 1728.
6. 72 and 180.
7. 192 and 460.
8. 105 and 168.
9. 364 and 512.
10. 450 and 750.

MULTIPLES.

140. A **Multiple** (dividend or product) of a number is any number of times that number; as 12, 18, 66 etc., are multiples of 6.

141. A **Common Multiple** of two or more numbers is a number that will contain each of those numbers without a remainder; thus 60 is a common multiple of 15, 10, 6 and 5, or of any two of them.

142. The **Least Common Multiple** of two or more numbers is the least number that will contain them without a remainder; thus 30 is the least common multiple of 10 and 15.

143. **Principles.**—*1. The product of two or more numbers, or any number of times their product, must be a common multiple of the numbers.*

2. Two or more numbers may have any number of common multiples.

3. A multiple of a number must contain all the prime factors of that number.

4. The least common multiple of two or more numbers must contain all the prime factors found in each of those numbers once and no oftener.

5. The least common multiple must be the product of the prime factors used the greatest number of times in any one number.

144. To find the least common multiple.

EXAMPLE.—Find the l. c. m. of 60, 150 and 270.

OPERATION.

3) 60 — 150 — 270
2) 20 — 50 — 90
5) 10 — 25 — 45
2 5 9

l. c. m. = $3^3 \times 2^2 \times 5^2 = 2700$

EXPLANATION.—Since the least common multiple must contain all the prime factors found in any one number *once* and *no oftener* (Princp. 4), therefore, dividing by 3 in the above operation cancels the 3's not used, as also by 2 and by 5. By factoring, 60 = 3×2×2×5; 150=3×2×5×5; 270=3×3×3×2×5. It will be seen that 3 is used the greatest number of times in 270 (3 times), 5 the greatest number of times in 150 (2 times), and 2 in 60 (2 times). Therefore the l. c. m. = 2×2×3×3×3×5×5.

Rule.—I. *Write the numbers in a horizontal line, separating them by a dash.*

II. *Divide by any factor common to all the numbers, or by any prime factor of any two or more of them. In the same manner divide the quotients obtained, and continue until the quotients are prime to each other.*

III. *The product of the divisors and prime quotients is the least common multiple.*

EXAMPLES.

145. Find the l. c. m. of

1. 32 and 88.
2. 34 and 60.
3. 57 and 95.
4. 81 and 117.
5. 36 and 150.
6. 12, 44, 60.
7. 44, 88, 100.
8. 3, 5, 8, 9, 10, 15.
9. 5, 6, 10, 12, 15.
10. 4, 8, 12, 16, 50.
11. 144, 1728, 60.
12. 56, 512, 128.
13. 11, 33, 66, 165.
14. 9, 18, 45, 63.
15. 14, 49, 77, 22.

The practical value of finding the least common multiple is only in determining the least common denominator of fractions.

CANCELLATION.

146. Cancellation is the omission of the same factor from numbers sustaining to each other the relation of *dividend* and *divisor*. It is a short method of division.

To cancel means to cross out, and cancellation means dividing by crossing out.

147. The **Sign of Cancellation** is a short line drawn obliquely through a figure; thus, $\cancel{2}$.

EXAMPLE.—Divide the product of 21 × 15 × 9 by the product of 5 × 7 × 3.

OPERATION.

$$\frac{\overset{3}{\cancel{21}} \times \overset{3}{\cancel{15}} \times \overset{3}{\cancel{9}}}{\cancel{5} \times \cancel{7} \times \cancel{3}}$$

$$3 \times 3 \times 3 = 27.$$

EXPLANATION.—The product of 21 × 15 × 9 is found by multiplication to be 2835, and the product of 5 × 7 × 3 is 105. Dividing 2835 by 105 gives 27 for a quotient. But as above indicated, the divisor, 105, is composed of the factors 5, 7, 3, and the dividend is composed of the factors 21, 15, 9. Now, instead of dividing 2835 by 105, divide the factors of one number by the factors of the other number. Commencing with 5 below the line, say 5 into 15 (one of the numbers above the line) 3 times. Draw lines through the 5 and the 15 to show that the division has been made by cancellation and write the quotient 3 above the number divided. Proceed in like manner with the remaining factors, and obtain 27 (3 × 3 × 3) as a final quotient.

When there is no uncancelled number in the dividend that will contain any uncancelled number in the divisor, divide by any factor common to both; thus, 24 will not contain 9, but each will contain 3, a factor common to both.

Rule.—I. ***Write the divisor below the dividend with a line separating them.***

II. ***Cancel from the dividend and divisor all factors common to both; then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.***

This principle can be put to frequent and valuable use in a great variety of business computations.

EXAMPLES.

148. *1.* Divide 5 × 17 × 28 × 36 × 9 by 34 × 14 × 18 × 3.

2. 81 × 25 × 34 × 30 ÷ by 21 × 5 × 6 × 17 = ?

3. Divide 24 × 25 × 26 × 27 by 2 × 4 × 5 × 9 × 13.

4. Divide 51 × 7 × 9 × 27 × 40 × 54 by 63 × 17 × 9 × 200.

5. Divide the product of 416, 216, and 810 by the product of 135, 52, 24, and 5.

6. How many chickens at 15 cents each must be given in exchange for 35 geese at 45¢ each?

7. How many bushels of potatoes at 60¢ per bushel will pay for 450 lb. of sugar at 6¢ per pound?

8. I bought 18 car loads of apples of 216 barrels each, each barrel containing 3 bushels at 60¢ per bushel, and paid for the same in woolen cloth. If each bale of cloth contained 600 yds. at 30 cents per yard, how many bales and how many odd yards did I deliver?

FRACTIONS.

149. A **Fraction** is one or more of the equal parts of a unit. If a unit be divided into 3 equal parts, one of the parts is called *one-third,* and is written $\frac{1}{3}$; two of the parts are called *two-thirds,* and are written $\frac{2}{3}$.

150. The **Terms** of a fraction are the *numerator* and *denominator,* taken together.

151. The **Numerator** is the number above the line; it numerates, or numbers the parts, and is a dividend.

152. The **Denominator** is the number below the line; it denominates, or names the value, or size, of the parts showing the number of parts into which the unit has been divided. It is a divisor.

153. The **Value** of a fraction is the *quotient* obtained by dividing the *numerator* by the *denominator.*

NOTATION AND NUMERATION.

154. Indicate by figures the following, and number and name the parts:

1. Four thirteenths.
2. Seventeen nineteenths.
3. Six sevenths.
4. Nine twelfths.
5. Six one hundredths.
6. 17 fortieths.
7. 83 ninety-ninths.
8. 19 twentieths.
9. 45 forty-eighths.
10. 14 fiftieths.
11. $\frac{13}{\text{fifteenths}}$
12. $\frac{49}{\text{eightieths}}$
13. $\frac{2}{\text{twenty-firsts}}$

155. Copy and read the following:

1. $\frac{3}{4}$.
2. $\frac{5}{6}$.
3. $\frac{2}{7}$.
4, $\frac{3}{5}$.
5. $\frac{1}{8}$.
6. $\frac{5}{9}$.
7. $\frac{8}{12}$.
8. $\frac{5}{13}$.
9. $\frac{6}{20}$.
10. $\frac{17}{45}$.
11. $\frac{15}{81}$.
12. $\frac{3}{100}$.
13. $\frac{2}{21}$.
14. $\frac{8}{25}$.
15. $\frac{9}{72}$.
16. $\frac{15}{3}$.
17. $\frac{48}{6}$.
18. $4\frac{1}{2}$.
19. $8\frac{3}{4}$.
20. $27\frac{9}{10}$.

REDUCTION.

156. **Reduction of Fractions** is the process of changing their form without changing their value.

157. To **Higher Terms.**—A fraction is reduced to higher terms when its numerator and denominator are expressed in larger numbers; thus, $\frac{4}{5}$ changed to higher terms = $\frac{24}{30}$.

158. To **Lower Terms.**—A fraction is reduced to lower terms when its numerator and denominator are expressed in lower numbers; thus, $\frac{24}{30}$ changed to lower terms = $\frac{12}{15}$.

159. To **Lowest Terms.**—A fraction is reduced to its lowest terms when its numerator and denominator are prime to each other; thus, $\frac{24}{30}$ changed to its lowest terms = $\frac{4}{5}$.

160. **How Changed.**—Fractions are changed to higher terms by multiplication, and to lower terms by division.

161. To change a fraction to higher terms.

Principle.—*Multiplying both terms of a fraction by the same number changes the form but does not alter the value of the fraction.*

Example.—Change \$$\frac{1}{2}$ to fourths of a dollar.

OPERATION.

$4 \div 2 = 2.$

$\$\frac{1 \times 2}{2 \times 2} = \$\frac{2}{4}.$

Explanation.—In $\frac{1}{2}$ of a dollar there are $\frac{2}{4}$ of a dollar. Dividing the required denominator 4 by the given denominator 2 gives 2 as a quotient. Multiplying both terms of the fraction by 2 gives $\frac{2}{4}$ or \$$\frac{2}{4}$, the required fraction.

In the above example changing one-half to two-fourths increases the number of parts taken, but decreases the value of the parts.

Rule.—I. *Multiply both terms of the fraction by such a number as will change the given denominator to the required denominator.*

II. *Such a number is found by dividing the required denominator by the given denominator.*

EXAMPLES.

162. Change

1. $\frac{2}{3}$ to 15ths.
2. $\frac{1}{5}$ to 15ths.
3. $\frac{2}{7}$ to 21sts.
4. $\frac{2}{3}$ to 21sts.
5. $\frac{5}{12}$ to 36ths.
6. $\frac{1}{3}$ to 36ths.
7. $\frac{5}{9}$ to 36ths.
8. $\frac{3}{9}$ to 27ths.
9. $\frac{3}{40}$ to 120ths.
10. $\frac{5}{8}$ to 88ths.
11. $\frac{7}{13}$ to 52nds.
12. $\frac{2}{23}$ to 115ths.
13. $\frac{3}{16}$ to 128ths.
14. $\frac{5}{24}$ to 192nds.
15. $\frac{11}{21}$ to 147ths.

163. To change a fraction to lower terms.

Principle.—*Dividing both terms of a fraction by the same number changes the form but does not alter the value of the fraction.*

EXAMPLE.—Change $\$\frac{75}{100}$ to fourths of a dollar.

OPERATION.

$100 \div 4 = 25.$

$\$\frac{75}{100} \div \frac{25}{25} = \$\frac{3}{4}.$

EXPLANATION.—In $\$\frac{75}{100}$ of a dollar there are $\frac{3}{4}$ of a dollar. Dividing the given denominator by the required denominator gives 25 as a quotient. Dividing both terms of the fraction by 25 gives $\frac{3}{4}$ or $\$\frac{3}{4}$, the required fraction.

In the foregoing example changing $\$\frac{75}{100}$ to $\$\frac{3}{4}$ *decreases* the number of parts, but increases the *value* of the parts.

Rule.—I. *Divide both terms of the fraction by such a number as will change the given denominator to the required denominator.*

II. *Such a number is found by dividing the given denominator by the required denominator.*

EXAMPLES.

164. Change

1. $\frac{10}{15}$ to thirds.
2. $\frac{3}{15}$ to fifths.
3. $\frac{6}{21}$ to sevenths.
4. $\frac{15}{21}$ to sevenths.
5. $\frac{15}{36}$ to twelfths.
6. $\frac{12}{36}$ to twelfths.
7. $\frac{15}{60}$ to twentieths.
8. $\frac{9}{27}$ to ninths.
9. $\frac{9}{120}$ to fortieths.
10. $\frac{55}{88}$ to eighths.
11. $\frac{28}{52}$ to thirteenths.
12. $\frac{10}{115}$ to twenty-thirds.
13. $\frac{24}{128}$ to sixteenths.
14. $\frac{40}{192}$ to twenty-fourths.
15. $\frac{77}{147}$ to twenty-firsts.
16. $\frac{126}{360}$ to sixtieths.

165. To reduce a fraction to its lowest terms

EXAMPLE.—Reduce $\frac{18}{24}$ to its lowest terms.

OPERATION.

$$\frac{18}{24} = \frac{\cancel{3} \times 3 \times \cancel{2}}{\cancel{3} \times 4 \times \cancel{2}} = \frac{3}{4}, \text{ or}$$

6 is the g. c. d. of 18 and 24.

$\frac{18}{24} \div \frac{6}{6} = \frac{3}{4}.$

EXPLANATION.—As a fraction is reduced to its lowest terms when its numerator and denominator are prime to each other, resolve the numerator 18 and the denominator 24 into their prime factors and cancel those common to both; or divide both terms by their greatest common divisor.

Rules.—*1. Divide both terms of the fraction by their greatest common divisor. Or,*

2. Divide by any prime number common to both terms,

and continue to divide by some prime number until the resulting numerator and denominator are prime to each other.

Two numbers are prime to each other when there are no factors in one found in the other. Thus, 6 and 25, although both are composite numbers, are prime to each other, as the factors of 6 (2 × 3) are prime to the factors of 25 (5 × 5).

EXAMPLES.

166. Reduce to their lowest terms:

1. $\frac{25}{40}$.
2. $\frac{8}{80}$.
3. $\frac{16}{40}$.
4. $\frac{27}{72}$.
5. $\frac{64}{80}$.
6. $\frac{42}{98}$.
7. $\frac{75}{100}$.
8. $\frac{81}{123}$.
9. $\frac{125}{240}$.
10. $\frac{48}{120}$.
11. $\frac{144}{1728}$.
12. $\frac{1728}{8640}$.

167. To reduce a mixed number to an improper fraction.

EXAMPLE.—Reduce $5\frac{2}{5}$ to an improper fraction.

OPERATION.

$5\frac{2}{5}$
$25 = 5 \times 5$, the denominator.
$2 =$ the numerator added.
$5\frac{2}{5} = \frac{27}{5}$.

EXPLANATION. — Since 1 unit equals 5 fifths, 5 units equals 5 times 5 fifths or 25 fifths; and 25 fifths plus 2 fifths equals 27 fifths; therefore $5\frac{2}{5} = \frac{27}{5}$.

Rule.—***Multiply the whole number by the denominator of the fraction, to the product add the numerator, and place the sum over the denominator.***

EXAMPLES.

168. Reduce to improper fractions:

1. \$$5\frac{2}{7}$.
2. $8\frac{3}{4}$ hours.
3. $5\frac{1}{2}$ tons.
4. $16\frac{2}{3}$ cents.
5. $12\frac{7}{8}$ pounds.
6. $75\frac{4}{5}$ cwt.
7. $16\frac{8}{9}$ M.
8. $1728\frac{11}{12}$.

169. To reduce an improper fraction to a whole or mixed number.

EXAMPLE.—Reduce $\frac{27}{5}$ to a mixed number.

OPERATION.

$\frac{27}{5} = 5\,)\,27\,(\,5\frac{2}{5}$.

EXPLANATION.—Since $\frac{5}{5}$ equals 1 unit, $\frac{27}{5}$ will equal as many units as 5 is contained in 27, or 5 times, with a remainder of 2 or $\frac{2}{5}$; therefore $\frac{27}{5} = 5\frac{2}{5}$.

Rule.—***Divide the numerator by the denominator, place the remainder, if any, over the denominator, and annex the fraction thus found to the entire part of the quotient.***

EXAMPLES.

170. Change to whole or mixed numbers:

1. $\$\frac{175}{25}$.
2. $\frac{145}{15}$ yards.
3. $\frac{115}{35}$ lbs.
4. $\frac{100}{3}$ cents.
5. $\frac{450}{125}$ cords.
6. $\frac{1728}{144}$ cu. feet.
7. $\frac{1885}{225}$ cwt.
8. $\frac{1012}{126}$ tons.
9. $\frac{854}{150}$ M lumber.
10. $\frac{1895}{125}$ inches.

171. To reduce fractions to equivalent fractions having a common denominator.

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$ to equivalent fractions having a common denominator.

OPERATION.

$\frac{2}{3} \times 4 \times 2 = \frac{16}{24}$.
$\frac{3}{4} \times 3 \times 2 = \frac{18}{24}$.
$\frac{1}{2} \times 3 \times 4 = \frac{12}{24}$.

EXPLANATION.—The product of the denominators, 3, 4 and 2 is 24, and this number is exactly divisible by each of the several denominators; hence, each fraction may be changed to equivalent fractions having the denominator 24, each separate denominator being a factor of the common denominator.

Rule.—*Multiply both terms of each fraction by the product of all the denominators except its own.*

EXAMPLES.

172. Reduce to equivalent fractions having a common denominator:

1. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{7}$.
2. $\frac{1}{2}$, $\frac{2}{9}$, $\frac{3}{5}$.
3. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{5}{7}$.
4. $\frac{17}{40}$, $\frac{7}{10}$, $\frac{1}{2}$, $\frac{1}{3}$.
5. $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$.
6. $3\frac{3}{4}$, $5\frac{2}{3}$, $14\frac{1}{2}$.
7. $17\frac{3}{4}$, $8\frac{1}{3}$, $6\frac{2}{3}$.
8. $21\frac{2}{6}$, $16\frac{3}{15}$, $19\frac{1}{2}$.
9. $8\frac{1}{2}$, $9\frac{1}{3}$, $10\frac{1}{4}$, $11\frac{1}{5}$.
10. $175\frac{3}{5}$, $75\frac{1}{2}$, $100\frac{2}{3}$.

173. To reduce to equivalent fractions having the least common denominator.

The least common denominator of two or more fractions is the least denominator to which they can be reduced, which must be the least common multiple of the given denominators.

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$ to equivalent fractions having the least common denominator.

OPERATION.

$12 =$ l. c. m. of 3, 4, 2.
$\frac{2}{3} \times 4 = \frac{8}{12}$.
$\frac{3}{4} \times 3 = \frac{9}{12}$.
$\frac{1}{2} \times 6 = \frac{6}{12}$.

EXPLANATION.—Find the l. c. m. of the given denominators which is 12. Then reduce each of the given fractions to a fraction whose denominator is 12.

Rule.—*Find the l. c. m. of the given denominators. Divide the least common denominator thus obtained by the denominator of each given fraction, and multiply its numerator by the quotient thus obtained.*

EXAMPLES.

174. Reduce to equivalent fractions having the least common denominator:

1. $\frac{2}{15}$, $\frac{47}{60}$, $\frac{5}{6}$, $\frac{17}{30}$.
2. $\frac{11}{48}$, $\frac{5}{96}$, $\frac{7}{12}$, $\frac{15}{24}$.
3. $\frac{4}{9}$, $\frac{8}{27}$, $\frac{17}{81}$, $\frac{5}{81}$.
4. $\frac{5}{36}$, $\frac{17}{144}$, $\frac{35}{72}$, $\frac{13}{18}$.
5. $\frac{3}{10}$, $\frac{121}{150}$, $\frac{63}{75}$, $\frac{13}{15}$.
6. $\frac{129}{256}$, $\frac{253}{512}$, $\frac{69}{128}$.
7. $8\frac{3}{4}$, $7\frac{5}{16}$, $3\frac{1}{32}$.
8. $13\frac{2}{3}$, $17\frac{1}{2}$, $19\frac{3}{4}$.
9. $12\frac{2}{5}$, $14\frac{2}{3}$, $18\frac{9}{15}$.
10. $\frac{5}{12}$, $\frac{37}{72}$, $\frac{1153}{1728}$, $\frac{93}{144}$.

ADDITION.

175. To add fractions having a common denominator.

Example.—Add $\frac{3}{17}$, $\frac{4}{17}$, $\frac{11}{17}$.

OPERATION.

$\frac{3}{17} + \frac{4}{17} + \frac{11}{17} = \frac{18}{17}$.
$\frac{18}{17} = 1\frac{1}{17}$.

Explanation.—Since the fractions have a common denominator, add the numerators and write the sum over the common denominator. As the result obtained is an improper fraction, change it to a mixed number.

Rule.—*Add the numerators and place the sum over the common denominators. If the result obtained be an improper fraction change it to a whole or mixed number.*

EXAMPLES.

176. Add

1. $\frac{2}{13}$, $\frac{3}{13}$, $\frac{5}{13}$, $\frac{4}{13}$.
2. $\frac{13}{20}$, $\frac{7}{20}$, $\frac{3}{20}$, $\frac{8}{20}$.
3. $\frac{39}{50}$, $\frac{27}{50}$, $\frac{37}{50}$, $\frac{11}{50}$.
4. $\frac{2}{150}$, $\frac{14}{150}$, $\frac{113}{150}$, $\frac{17}{150}$.
5. $\frac{71}{72}$, $\frac{3}{72}$, $\frac{19}{72}$, $\frac{51}{72}$.
6. $\frac{89}{144}$, $\frac{93}{144}$, $\frac{101}{144}$, $\frac{6}{144}$.

177. To add fractions having different denominators.

Example.—Find the sum of $\frac{2}{5}$, $\frac{7}{10}$, $\frac{1}{2}$.

OPERATION.

$\frac{2}{5} + \frac{7}{10} + \frac{1}{2} =$
$\frac{4}{10} + \frac{7}{10} + \frac{5}{10} = \frac{16}{10}$.
$\frac{16}{10} = 1\frac{6}{10} = 1\frac{3}{5}$.

Explanation.—Since only fractions having the same denominator can be added, reduce the fractions to their least common denominator, and writing their equivalents below, add their numerators and place the sum over the common denominator. Reduce the improper fraction thus obtained to a mixed number and express the fraction in its lowest terms.

Rule.—*Reduce the fractions to their least common denominator, add the resulting numerators, place the sum obtained over the common denominator and if necessary reduce the fraction.*

EXAMPLES.

178. Add

1. $\frac{3}{4}$, $\frac{2}{3}$, $\frac{4}{5}$.
2. $\frac{9}{10}$, $\frac{5}{8}$, $\frac{3}{20}$.
3. $\frac{5}{7}$, $\frac{2}{3}$, $\frac{2}{5}$.
4. $\frac{5}{8}$, $\frac{4}{9}$, $\frac{3}{4}$.
5. $\frac{3}{20}$, $\frac{5}{80}$, $\frac{7}{40}$.
6. $\frac{61}{81}$, $\frac{8}{9}$, $\frac{22}{27}$.
7. $\frac{139}{144}$, $\frac{1}{2}$, $\frac{2}{3}$.
8. $\frac{5}{12}$, $\frac{7}{18}$, $\frac{13}{36}$.
9. $\frac{7}{30}$, $\frac{11}{60}$, $\frac{5}{12}$.
10. $\frac{13}{16}$, $\frac{3}{8}$, $\frac{5}{32}$.
11. $\frac{5}{95}$, $\frac{3}{19}$, $\frac{4}{5}$.
12. $\frac{1}{26}$, $\frac{1}{65}$, $\frac{1}{39}$.
13. $\frac{53}{77}$, $\frac{76}{88}$, $\frac{89}{99}$.
14. $\frac{18}{105}$, $\frac{17}{35}$, $\frac{69}{70}$.
15. $\frac{3}{64}$, $\frac{5}{32}$, $\frac{15}{16}$.

179. To add mixed numbers.

EXAMPLE.—Find the sum of $8\frac{1}{2}$, $4\frac{2}{3}$, $5\frac{1}{4}$.

OPERATION.

$$\begin{array}{r}
8\frac{1}{2} = 8\frac{6}{12} \\
4\frac{2}{3} = 4\frac{8}{12} \\
5\frac{1}{4} = 5\frac{3}{12} \\
\hline
17\frac{17}{12} \\
\hline
1\frac{5}{12} \\
\hline
18\frac{5}{12}.
\end{array}$$

EXPLANATION.—Arrange the numbers to be added in a perpendicular line. Adding the whole numbers a result of 17 is obtained. Changing the fractional parts to equivalent fractions having a least common denominator and $\frac{17}{12}$ is their sum $= 1\frac{5}{12}$, which added to 17 gives $18\frac{5}{12}$.

Rule.—*Arrange the numbers to be added in perpendicular columns, add the integers, and to the sum thus obtained add the sum of the several fractions.*

Change the fractional parts to equivalent fractions having a common denominator according to Article 171.

EXAMPLES.

180. Add

1. $130\frac{3}{7}$, $69\frac{5}{8}$, $600\frac{6}{11}$, and 46.
2. $80\frac{1}{3}$, $2\frac{1}{3}$, $5\frac{2}{3}$, $41\frac{1}{6}$, and $14\frac{2}{5}$.
3. $28\frac{4}{7}$, $85\frac{1}{17}$, $60\frac{5}{11}$, and 11.
4. $47\frac{7}{8}$, $3\frac{5}{8}$, 4, $29\frac{2}{5}$, $66\frac{3}{13}$ and 4.
5. $16\frac{2}{3}$, $33\frac{1}{3}$, $66\frac{2}{3}$, $88\frac{1}{3}$, and $116\frac{2}{3}$.
6. $18\frac{3}{4}$, $65\frac{3}{5}$, $161\frac{3}{4}$, $67\frac{5}{8}$, and 75.

7. The six fields of a farm measure respectively 10, $12\frac{1}{2}$, $19\frac{1}{4}$, $26\frac{41}{160}$, $39\frac{17}{80}$, and $2\frac{1}{2}$ acres. How many acres in the farm?

8. Five carloads of coal weigh respectively $18\frac{3}{4}$, $16\frac{1}{9}$, $14\frac{2}{3}$, $17\frac{1}{2}$, $15\frac{5}{6}$ tons. Find total in tons.

9. Bot. of Goodyear & Co. 5 pcs. New York Mills Sheeting, measuring respectively, $41\frac{3}{4}$, $40\frac{7}{8}$, $39\frac{1}{4}$, $42\frac{5}{8}$, $44\frac{3}{8}$ yards. Find total in yards.

10. The five yards of Babcock & Co., coal dealers, inventory soft coal on hand as follows: $183\frac{19}{20}$, $142\frac{1}{2}$, $67\frac{14}{15}$, $28\frac{4}{5}$, $198\frac{16}{25}$ tons, and valued respectively \$$915\frac{3}{4}$, \$$710\frac{1}{4}$, \$$271\frac{11}{15}$, \$$144\frac{1}{20}$, \$$951\frac{1}{6}$. How many tons of coal in the yards and what is the total value.

SUBTRACTION.

181. To subtract fractions having different denominators.

EXAMPLE.—From $\frac{4}{5}$ take $\frac{2}{3}$.

OPERATION.

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

EXPLANATION.—Since only fractions having a common denominator can be subtracted, change the fractions to equivalent fractions having a common denominator, take the difference between the numerators, 12 and 10, and write this difference, 2, over the common denominator 15.

Rule.—I. *Change the given fractions to equivalent fractions having the least common denominator.*

II. *Subtract the numerator of the subtrahend from the numerator of the minuend and write the result over the common denominator.*

EXAMPLES.

182. Find the difference between

1. $\frac{1}{3}$ and $\frac{5}{6}$.
2. $\frac{3}{4}$ and $\frac{2}{3}$.
3. $\frac{4}{5}$ and $\frac{5}{7}$.
4. $\frac{5}{16}$ and $\frac{3}{4}$.
5. $\frac{6}{7}$ and $\frac{7}{8}$.
6. $\frac{53}{40}$ and $\frac{19}{60}$.
7. $\frac{53}{40}$ and $\frac{19}{20}$.
8. $\frac{5}{6}$ and $\frac{7}{9}$.
9. $\frac{17}{20}$ and $\frac{19}{30}$.
10. $\frac{1}{2}$ and $\frac{3}{50}$.

183. To subtract mixed numbers.

EXAMPLE.—From $18\frac{2}{3}$ take $12\frac{3}{4}$.

OPERATION.

$$\begin{array}{l} 18\frac{2}{3} = 17\frac{20}{12} \\ \underline{12\frac{3}{4}} = \underline{12\frac{9}{12}} \\ \qquad\quad\ \ 5\frac{11}{12} \end{array}$$

EXPLANATION.—Since $\frac{3}{4}$ cannot be taken from $\frac{2}{3}$, change $18\frac{2}{3}$ to $17\frac{5}{3}$ and then from $17\frac{5}{3}$ subtract $12\frac{3}{4}$, after changing the fractions to a common denominator.

In case the minuend is a whole number change 1 to fractional form; as, $8 - \frac{2}{3} = 7\frac{3}{3} - \frac{2}{3} = 7\frac{1}{3}$.

Rule.—I. *Write the subtrahend beneath the minuend.*

II. *Reduce the fractional parts to like denominators.*

III. *Subtract fractional and integral parts separately and unite the results.*

EXAMPLES.

184. From

1. $25\frac{3}{4}$ take $16\frac{1}{2}$.
2. $83\frac{1}{2}$ take $19\frac{7}{8}$.
3. 150 take $13\frac{1}{3}$.
4. $144\frac{1}{8}$ take $50\frac{3}{4}$.
5. $25\frac{17}{30}$ take $15\frac{1}{3}$.
6. $51\frac{1}{3}$ take $18\frac{3}{4}$.
7. 75 take $11\frac{3}{5}$.
8. $62\frac{11}{15}$ take $8\frac{1}{5}$.
9. $72\frac{3}{11}$ take $16\frac{8}{11}$.
10. 195 take $101\frac{4}{17}$.

MULTIPLICATION.

185. To multiply a fraction by a whole number.

Principle.—*1. Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE 1.—Multiply $\frac{3}{5}$ by 6.

OPERATION.

$\frac{3}{5} \times 6 = \frac{18}{5} = 3\frac{3}{5}$.

EXPLANATION.—Since the numerator is the number of parts taken, and as multiplying the numerator multiplies the fraction, multiply 3 by 6 and obtain 18 or $\frac{18}{5} = 3\frac{3}{5}$.

EXAMPLE 2.—Multiply $\frac{5}{8}$ by 4.

OPERATION.

$\frac{5}{8} \times 4 = \frac{20}{8} = 2\frac{1}{2}$, or

$\frac{5}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{1} = \frac{5}{2} = 2\frac{1}{2}$.

EXPLANATION.—Proceed as in the above example, or reduce by cancellation, which is equivalent to reducing a fraction to its lowest terms.

Rule.—*Multiply the numerator or divide the denominator, by the whole number.*

Observe that the numerator can always be multiplied, while the denominator can only be divided when the multiplier is a factor of it. When possible economize by cancellation and thus reduce the fraction to its simplest form.

EXAMPLES.

186. Find the products of

1. $\frac{3}{8}$ multiplied by 5.
2. $\frac{5}{12}$ multiplied by 6.
3. $\frac{5}{19}$ multiplied by 12.
4. $\frac{11}{15}$ multiplied by 3.
5. $\frac{7}{16}$ multiplied by 7.
6. $\frac{5}{18}$ multiplied by 9.
7. $\frac{17}{24}$ multiplied by 8.
8. $\frac{7}{50}$ multiplied by 25.
9. $\frac{89}{144}$ multiplied by 12.
10. $\frac{15}{165}$ multiplied by 15.

187. To multiply a whole number by a fraction.

EXAMPLE.—Multiply 18 by $\frac{2}{3}$.

OPERATION.

$\frac{1}{3}$ of 18 = 6

$\frac{2}{3}$ of 18 = 2 × 6 = 12

or $\frac{2}{\cancel{3}} \times \frac{\overset{6}{\cancel{18}}}{1} = 12.$

EXPLANATION.—To multiply 18 by $\frac{2}{3}$ is to get $\frac{2}{3}$ of 18. As $\frac{1}{3}$ of 18 is 6, $\frac{2}{3}$ of 18 = 2 × 6 or 12. Again by multiplying 18 by the numerator 2 the quotient is 3 times too large, as the multiplier is 2 divided by 3; hence divide by the denominator 3 which completes the operation indicated.

Since 18 and $\frac{2}{3}$ are the factors of the product, reverse the orders of the product and proceed as in Art. 185, and cancel when possible.

Rule.—*Multiply the whole number by the numerator of the fraction, and divide the product by the denominator. Cancel when possible.*

EXAMPLES.

188. Find the products of

1. 5 multiplied by $\frac{4}{5}$.
2. 7 multiplied by $\frac{3}{8}$.
3. 15 multiplied by $\frac{3}{5}$.
4. 17 multiplied by $\frac{3}{35}$.
5. 18 multiplied by $\frac{3}{7}$.
6. 42 multiplied by $\frac{6}{7}$.
7. 18 multiplied by $\frac{2}{3}$!
8. 21 multiplied by $\frac{5}{3}$.
9. 19 multiplied by $\frac{175}{95}$.
10. 13 multiplied by $\frac{7}{143}$.

Entire or mixed numbers may be treated with facility by reducing them to fractional forms and cancelling when possible.

189. To multiply a fraction by a fraction.

EXAMPLE.—Multiply $\frac{4}{5}$ by $\frac{2}{3}$.

OPERATION.

$\frac{1}{3}$ of $\frac{4}{5}$ = $\frac{4}{15}$

$\frac{2}{3}$ of $\frac{4}{5}$ =

2 × $\frac{4}{15}$ = $\frac{8}{15}$.

EXPLANATION.—To multiply $\frac{4}{5}$ by $\frac{2}{3}$ is to take $\frac{2}{3}$ of $\frac{4}{5}$. To get one-third, divide $\frac{4}{5}$ by 3, which equals $\frac{4}{15}$. As $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$, $\frac{2}{3} = 2 \times \frac{4}{15} = \frac{8}{15}$.

Rule.—I. *Indicate the operation and cancel all factors common to both numerator and denominator.*

II. *Multiply together the remaining numerators for the numerator of the product, and the remaining denominators for the denominator of the product.*

EXAMPLES.

190. Find the product of

1. $\frac{3}{5}$ of $\frac{9}{15}$
2. $\frac{5}{7}$ of $\frac{21}{35}$.
3. $\frac{3}{7}$ of $\frac{14}{33}$.
4. $\frac{3}{8}$ of $\frac{24}{75}$.
5. $\frac{1}{2}$ of $\frac{19}{93}$.
6. $\frac{4}{11}$ of $\frac{44}{75}$.
7. $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{4}{5}$.
8. $\frac{2}{3}$ of $\frac{6}{7}$ by $\frac{7}{18}$.
9. $\frac{3}{4}$ of $\frac{12}{17}$ by $\frac{34}{39}$.

191. To multiply a mixed number by a mixed number.

EXAMPLE.—Multiply $17\frac{4}{5}$ by $20\frac{1}{2}$.

OPERATION.

$$\begin{array}{l} 17\frac{4}{5} \\ \underline{20\frac{1}{2}} \\ 8\frac{1}{2} = 17 \times \frac{1}{2} \\ 16 \ = 20 \times \frac{4}{5} \\ \ \ \frac{2}{5} = \frac{4}{5} \text{ of } \frac{1}{2} \\ \underline{340 \ = 17 \times 20} \\ 364\frac{9}{10} \end{array}$$

EXPLANATION.—$\frac{1}{2}$ of $17 = 8\frac{1}{2}$; $\frac{4}{5}$ of $20 = 16$; $\frac{1}{2}$ of $\frac{4}{5} = \frac{2}{5}$; $17 \times 20 = 340$; $8\frac{1}{2} + 16 + \frac{2}{5} + 340 = 364\frac{9}{10}$.

Rule.—*Arrange the factors as in multiplication of whole numbers. Multiply together the integral parts, and to this product add the product of the integral part of each factor multiplied by the fractional part of the other factor; also, if each factor be a mixed number add the product of the fractional part of one factor multiplied by the fractional part of the other factor.*

BUSINESS EXAMPLES.

192. *1.* Find the cost of $84\frac{3}{4}$ cords of wood at $8.50 per cord.

2. Sold Morgan, Ransom & Co. for cash, $75\frac{4}{5}$ tons Blossburg coal at $\$5\frac{2}{3}$ per ton.

3. Mrs. Marion Ensign, bought of C. B. Blood & Co., $24\frac{3}{4}$ yds. velvet at $\$6\frac{2}{3}$ per yard. Find amount of invoice.

4. Bot. of Gould & Co., $15\frac{4}{5}$ yds. broadcloth at $\$4\frac{2}{5}$, $16\frac{2}{5}$ yds. velvet at $\$5\frac{3}{4}$. Paid for same as follows: $25\frac{1}{2}$ cords wood at $\$3\frac{4}{5}$ per cord, $9\frac{3}{4}$ tons coal at $\$4\frac{2}{3}$ per ton, and cash for balance. How much cash was paid?

5. Thos. W. Cooper's inventory book, Jan. 1, 1891, shows the following:

188	tons egg coal,	$\$4\frac{3}{4}$.	$150\frac{4}{5}$	cords wood,	$\$5\frac{2}{3}$.
75	tons nut coal,	$\$4\frac{1}{2}$.	$95\frac{3}{4}$	M lumber,	$\$12\frac{4}{5}$.
$60\frac{3}{4}$	tons stove coal,	$\$4\frac{4}{5}$.	$44\frac{2}{3}$	C shingles,	$\$5\frac{1}{3}$.

Find the total amount.

DIVISION.

193. To divide a fraction by a whole number.

Principle.—*Dividing the numerator or multiplying the denominator divides the fraction.*

EXAMPLE 1.—Divide $\frac{4}{9}$ by 2.

OPERATION.

$\frac{4}{9} \div 2 = \frac{2}{9}$

EXPLANATION.—By the general principles of fractions dividing the numerator divides the fraction ; hence, divide the numerator 4 of the fraction $\frac{4}{9}$ by 2 and the quotient is $\frac{2}{9}$.

EXAMPLE 2.—Divide $\frac{10}{17}$ by 3.

OPERATION.

$\frac{10}{17} \times 3 = \frac{10}{51}$.

EXPLANATION.—By the general principles of fractions multiplying the denominator divides the fraction; hence, when the numerator will not contain the divisor, multiply the denominator. Therefore $\frac{10}{17} \times 3 = \frac{10}{51}$.

EXAMPLE 3.—Divide $2815\frac{2}{3}$ by 8.

OPERATION.

$$\begin{array}{r} 8\underline{)\,2815\frac{2}{3}} \\ 351\frac{23}{24} \end{array}$$

EXPLANATION.—(Short Method).—Write as in short division and divide ; 8 is contained in $2815\frac{2}{3}$, 351 times with a remainder of $7\frac{2}{3}$ not divided ; reduce the 7 to thirds and to the result add the $\frac{2}{3}$ making $\frac{23}{3}$, which divide by 8, obtaining $\frac{23}{24}$ as the fractional part of the quotient; annex this to the integral part which gives $351\frac{23}{24}$.

Rule.—*Divide the numerator when it can be divided without a remainder. If the numerator cannot be divided without a remainder, multiply the denominator.*

EXAMPLES.

194. Divide

1. $\frac{34}{41}$ by 17.
2. $\frac{35}{61}$ by 7.
3. $\frac{204}{351}$ by 12.
4. $\frac{75}{91}$ by 15.
5. $150\frac{3}{5}$ by 6.
6. $75\frac{11}{15}$ by 15.
7. $144\frac{36}{51}$ by 12.
8. $185\frac{5}{9}$ by 9.
9. $205\frac{5}{6}$ by 6.
10. $108\frac{1}{3}$ by 5.
11. $1729\frac{1}{12}$ by 12.
12. $1950\frac{2}{3}$ by 7.

195. To divide a whole humber by a fraction.

EXAMPLE.—Into how many pieces, $\frac{2}{3}$ of a yard each, may 5 yards of ribbon be cut ?

OPERATION.

$5 \div \frac{2}{3} =$
$\frac{15}{3} \div \frac{2}{3} = 7\frac{1}{2}$.

EXPLANATION.—Since 5 yds. equals $\frac{15}{3}$ yd. they may be cut into as many pieces, each containing $\frac{2}{3}$ yd., as $\frac{2}{3}$ is contained times in $\frac{15}{3}$ which is $7\frac{1}{2}$ times.

Since the denominator *names* or tells the *kind*, or value of the parts taken, when fractions are reduced to the same denomination, or to equivalent fractions having a common denominator, their numerators compare as whole numbers. We may consequently ignore the denominators.

Rule.—*Multiply the denominator of the fraction by the whole number, and divide the result by the numerator.*

EXAMPLES.

196. Divide

1. 17 by $\frac{3}{7}$.
2. 11 by $\frac{5}{9}$.
3. 20 by $\frac{14}{15}$.
4. 86 by $\frac{5}{3}$.
5. 101 by $\frac{10}{88}$.
6. 24 by $\frac{3}{5}$.
7. 30 by $\frac{1}{10}$.
8. 12 by $\frac{5}{6}$.
9. 33 by $\frac{11}{12}$.
10. 15 by $\frac{5}{7}$.
11. 14 by $\frac{14}{20}$.
12. 20 by $\frac{9}{10}$.
13. 51 by $\frac{25}{33}$.
14. 39 by $\frac{11}{15}$.
15. 25 by $\frac{61}{7}$.

197. To divide a fraction by a fraction.

EXAMPLE.—Divide $\frac{2}{3}$ by $\frac{3}{4}$.

OPERATION.

$\frac{2}{3} \div \frac{3}{4} =$
$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

EXPLANATION.—The divisor $\frac{3}{4}$ is 3 ÷ 4 or $\frac{1}{4}$ of 3. First divide the dividend $\frac{2}{3}$ by 3 by multiplying its denominator (Art. 193) and the quotient is $\frac{2}{9}$. But 3 is 4 times greater than the divisor $\frac{3}{4}$, hence the quotient of $\frac{2}{3}$ ÷ 3 or $\frac{2}{9}$ is too small. Since the divisor is $\frac{1}{4}$ of 3 (3 ÷ 4 or $\frac{3}{4}$) complete the operation indicated and multiply $\frac{2}{9}$ by 4 which equals $\frac{8}{9}$.

Rule.—*Invert the terms of the divisor and proceed as in multiplication of fractions.*

Reduce mixed numbers to improper fractions before applying the rule.

EXAMPLES.

198. Divide

1. $\frac{19}{90}$ by $\frac{25}{49}$.
2. $\frac{41}{10}$ by $\frac{2}{3}$.
3. $\frac{51}{60}$ by $\frac{17}{20}$.
4. $\frac{35}{36}$ by $\frac{105}{19}$.
5. $\frac{240}{11}$ by $\frac{40}{33}$.
6. $\frac{25}{26}$ by $\frac{5}{13}$.
7. $\frac{3}{17}$ by $\frac{5}{19}$.
8. $\frac{21}{65}$ by $\frac{7}{13}$.
9. $\frac{95}{1728}$ by $\frac{19}{144}$.
10. $\frac{256}{930}$ by $\frac{128}{310}$.
11. $5\frac{2}{5}$ by $1\frac{3}{4}$.
12. $12\frac{7}{8}$ by $3\frac{2}{3}$.

MISCELLANEOUS EXAMPLES.

199. *1.* From $16\frac{3}{4} + 1\frac{5}{6}$ take $6\frac{9}{11} \times 1\frac{2}{3}$.

2. Multiply $(2\frac{1}{2} \div \frac{3}{4})$ by $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{11}{13}$.

3. The net gain of the firm of Jones, Lee & Emerson is $2760. Jones receives $\frac{3}{5}$ of it, Lee $\frac{2}{5}$ as much as Jones, and Emerson the remainder. What fractional part represents Emerson's share and what is the amount?

4. A jobber bought a stock of goods for $1575.50 and sold $\frac{5}{9}$ of it at an advance of $\frac{1}{7}$ of first cost, and the remainder at a loss of $\frac{1}{10}$ of first cost. Did he make or lose and how much.

5. If A and B working together equally can do a piece of work in 20 days of 9 hours each, in how many days of 12 hours each can A do it working alone.

6. A cistern has two faucets. The first will empty it if left open in $9\frac{1}{2}$ hours, the second, in $11\frac{3}{4}$ hours. In how many hours will the cistern be emptied if both faucets are left open?

7. A farmer left an estate of $3575, to be divided as follows: to his wife $\frac{2}{5}$ of the estâte, and to his three sons the remainder in the proportion of 1, 2 and 3. What was the portion of each?

8. John Morgan, Peter Cowan and Myron Smith do a certain job of work for $360. How should the money be divided, if divided in the proportion respectively of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

To compare fractions change to equivalent fractions having a common denominator and then compare the numerators.

9. In an orchard $\frac{3}{5}$ of the trees are apple trees; $\frac{1}{2}$ of the remainder, peach trees; $\frac{1}{4}$ of the remainder are pear trees, and the remainder are cherry trees. If the number of the cherry trees is 60, what was the total cost of the trees when set out, the apples costing $12\frac{1}{2}$¢ each, the peaches $18\frac{3}{4}$¢ each, pears $37\frac{1}{2}$¢ each, and cherries $16\frac{2}{3}$¢ each.

10. A manufacturer finds that the cost of the raw material for a dozen gloves is \$$8\frac{3}{4}$. To this he adds $\frac{1}{8}$ of the cost for manufacturing expenses, $\frac{1}{25}$ of the cost for interest, $\frac{3}{175}$ for boxing and $\frac{1}{5}$ of the cost for profit. If the cutting and making which costs \$$2.90\frac{5}{8}$ be added, what is the selling price?

11. How many suits of clothes containing $7\frac{3}{8}$ yards each can be cut from a piece of broadcloth containing $110\frac{5}{8}$ yards.

12. Find the total number of bushels that can be put in 5 bins based upon the following:

$\frac{2}{3}$ of 1st bin equals 150 bushels,
$\frac{17}{20}$ of 2nd bin equals 850 bushels,
$\frac{5}{19}$ of 3rd bin equals 95 bushels,
$\frac{3}{15}$ of 4th bin equals 120 bushels,
$\frac{13}{39}$ of 5th bin equals 260 bushels.

DECIMALS.

200. A **Decimal Fraction** is a fraction whose denominator is 10 or some multiple of ten; as, $\frac{3}{10}$, $\frac{4}{100}$, $\frac{7}{1000}$, $\frac{16}{10000}$.

The two points of difference between *common* and *decimal* fractions are,

1.—The denominator of a common fraction is always written, while that of a decimal is only indicated.

2. The denominator of a common fraction may be any number, while that of a decimal must be 10 or some multiple of 10.

201. The **Decimal Point** is a period (.). It is always placed at the left of tenths, and by its position it indicates the *denominator* and determines the *value* of the fraction; as .4, .35, .575.

When the decimal point is used to separate the integral from the fractional part in mixed decimals, or dollars and cents in decimal currency, it is called a *separatrix;* as 3.75, $9.25.

202. A **Pure Decimal** corresponds to a proper fraction, the value being less than one; as, .3, .17, .125.

203. A **Mixed Decimal** corresponds to an improper fraction, the value being greater than one; as, 8.5, 17.8, 18.185.

204. Principles.—*1. Decimals increase in value from right to left, and decrease from left to right, in a tenfold ratio.*

2. A decimal should contain as many places as there would be ciphers in its denominator if written, the decimal point representing the unit 1 of such denominator.

3. The value of any decimal figure depends upon its place from the decimal point.

4. Prefixing a cipher to a decimal decreases its value the same as dividing it by ten.

5. Annexing one or more ciphers to a decimal does not alter its value.

NUMERATION.

205. The **Order of a Decimal** may be found by numerating either from *right* to left or from *left* to right, only let it be remembered that the *decimal point* stands in the position of the *unit* 1 of the decimal denominator.

The value of a decimal may be determined by the same numeration as that employed in integers.

The relation of orders in a mixed decimal is clearly shown by the following

TABLE.

Hundreds of millions.	Tens of millions.	MILLIONS.	Hundreds of thousands.	Tens of thousands.	THOUSANDS.	Hundreds.	Tens.	UNITS.	Decimal Point.	Tenths.	Hundredths.	THOUSANDTHS.	Ten-thousandths.	Hundred-thousandths.	MILLIONTHS.	Ten-millionths.	Hundred-millionths.
9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.
1	1	1	1	1	1	1	1	1	.	1	1	1	1	1	1	1	1
The Integral Part.										*The Fractional Part.*							

The above number is read 111 million 111 thousand 111, and 11 million 111 thousand 111 *hundred-millionths.*

It is better, in reading *mixed* decimals, to connect the integral and fractional parts by *and;* as, 2.5 read 2 and 5 tenths; 17.016, 17 and 16 thousandths.

206. The **Denominator** of a decimal fraction is understood, and is 1 with as many ciphers annexed as there are decimal places in the decimal; as, $.8 = \frac{8}{10}$; $.17 = \frac{17}{100}$; $.08 = \frac{8}{100}$; $.125 = \frac{125}{1000}$.

207. To read decimals.

Rule.—I. *Numerate from the decimal point, to determine the denominator.*

II. *Read the decimal as a whole number, and give to it the denomination of the right-hand figure.*

EXAMPLES.

208. Read the following decimals:

1. .8.
2. .17.
3. .125.
4. .6421.
5. .08.
6. .017.
7. .3.
8. .30.
9. .300.
10. .3000.
11. .03.
12. .003.
13. .0003.
14. 45.45.
15. 185.185.
16. 175.125.
17. 2006.18.
18. 302.000402.

NOTATION.

209. In writing decimals keep in mind the following facts:

1st. That they are fractions.

2nd. That both the numerator and the denominator should be written or indicated.

3rd. That the denominator of any decimal (if written) is 1 with as many ciphers to the right as there are decimal places indicated in the numerator.

4th. When the numerator of the decimal does not contain as many places as the denominator (if written) would contain ciphers, prefix ciphers to make the number of places equal.

210. To write decimals.

Example.—Write as a decimal *three-tenths.*

Explanation.—Observe that in writing three-tenths as a common fraction, the mental operation is as follows: after writing 3, the numerator, you ask yourself 3 *what?* the answer is, 3 *tenths;* then the ten is written below as a denominator, thus obtaining $\frac{3}{10}$. Now reason in the same way regarding the decimal, and after writing 3, the numerator, ask yourself 3 *what?* and answer, 3 *tenths;* and *indicate* it by placing before the 3, a decimal point, which represents the 1 of the decimal denominator; notice that the 3 occupies one place corresponding to the one cipher in the denominator.

By extending and developing this method of writing decimals, the pupil can in a short time master the entire matter, so that he can write *any* decimal as readily and with as great certainty as if it were a whole number.

Rule.—I. *Write the decimal the same as a whole number, prefixing ciphers when necessary, to give to each figure its true local value.*

II. *Place the decimal point before the left-hand figure of the decimal.*

EXAMPLES.

211. Express by figures the following decimals:

1. Twenty-six thousandths.
2. Twenty-seven hundredths.
3. Sixteen ten-thousandths.
4. Four hundredths.
5. Twenty-two hundred-thousandths.
6. Five and seven-tenths.

7. Eighty-three and five hundred four ten-thousandths.

8. Seven hundred ten and two hundred forty-three hundred thousandths.

9. Five hundred and five hundredths.

10. Forty-five and forty-six thousandths.

212. Write as decimals the following:

1. $30\frac{75}{100}$.
2. $\frac{3075}{100}$.
3. $8\frac{25}{100}$.
4. $\frac{825}{100}$.
5. $\frac{563}{100}$.
6. $3\frac{815}{1000}$.
7. $\frac{2815}{1000}$.
8. $\frac{508}{100}$.
9. $5\frac{8}{100}$.
10. $\frac{1001}{1000}$.
11. $\frac{10007}{1000}$.
12. $\frac{150015}{10000}$.

13. Write 517 *tenths;* 517 *hundredths;* 517 *thousandths.*

14. Thirty-five hundred six *tenths;* thirty five hundred six *hundredths;* thirty-five hundred six *thousandths;* thirty-five hundred six *ten thousandths;* thirty-five hundred six *hundred thousandths.*

REDUCTION.

213. To reduce decimals to common fractions.

EXAMPLE 1.—Reduce .9 to a common fraction.

OPERATION.
$.9 = \frac{9}{10}$

EXPLANATION.—The decimal .9 is read *nine-tenths,* which as a common fraction is written $\frac{9}{10}$. Also, since the decimal has one place its indicated denominator must be 10.

EXAMPLE 2.—Reduce .16 to a common fraction.

OPERATION.
$.16 = \frac{16}{100}$
$\frac{16}{100} \div \frac{4}{4} = \frac{4}{25}$

EXPLANATION.—The two decimal places indicate the denominator to be *10* used as a factor *twice* or 100; $\frac{16}{100}$ reduced to its lowest terms equals $\frac{4}{25}$.

Rule.—*Omit the decimal point in the numerator and write for a denominator 1 with as many ciphers annexed as there are decimal places in the decimal.*

Mixed decimals are reduced in a similar manner.

EXAMPLES.

214. Reduce to common fractions in their lowest terms

1. .3.
2. .8.
3. .25.
4. .75.
5. .375.
6. .625.
7. .875.
8. .285.
9. .5856.
10. 16.75.
11. 125.48.
12. 24.24.
13. $.12\frac{1}{2}$.
14. $.6\frac{1}{4}$.
15. $.37\frac{1}{2}$.
16. $.33\frac{1}{3}$.
17. $.8\frac{2}{3}$.
18. $.025\frac{1}{4}$.

215. To reduce common fractions to decimals.

EXAMPLE.—Change $\frac{2}{5}$ to an equivalent decimal.

OPERATION.

$$\begin{array}{r} 5\overline{)2.0} \\ \hline .4 \end{array}$$

EXPLANATION.—To change a fraction to a decimal is to change it to a fraction whose denominator is *ten* or some multiple of ten. Since $\frac{2}{5} = \frac{1}{5}$ of 2, change 2 to *tenths* and $\frac{1}{5}$ of 20 *tenths* equals 4 *tenths*.

Rule.—*Place a decimal point and ciphers at the right of the numerator; divide by the denominator, and from the right of the quotient point off for decimals as many places as there have been ciphers used.*

To change a common fraction to a pure decimal, the denominator of the common fraction must contain the prime factors 2 or 5, and no others. If there are other prime factors than 2 or 5 the quotient will not be a complete decimal; as, $\frac{2}{7}$ changed to a decimal = .2857$\frac{1}{7}$, carried out to 4 decimal places; $\frac{1}{3}$ = .3$\frac{1}{3}$, or .33$\frac{1}{3}$, or .333$\frac{1}{3}$, according to the number of decimal places used, with a fractional remainder in each. These are called circulating or repeating decimals, and the figures repeated are called repetends.

EXAMPLES.

216. Reduce to equivalent decimals

1. $\frac{1}{2}$.
2. $\frac{3}{4}$.
3. $\frac{4}{5}$.
4. $\frac{5}{8}$.
5. $\frac{7}{8}$.
6. $\frac{5}{9}$.
7. $\frac{5}{16}$.
8. $\frac{1}{3}$.
9. $\frac{2}{3}$.
10. $\frac{1}{16}$.
11. $\frac{5}{12}$.
12. $\frac{113}{500}$.
13. $\frac{12}{25}$.
14. $\frac{1}{64}$.
15. $\frac{3}{12500}$.

Carry the results out to four decimal places.

ADDITION.

217. To add decimals.

EXAMPLE.—Add .7, 2.43, .865, 11.5, 113.2075, 200.00165.

OPERATION.

$$\begin{array}{r@{.}l} & 7 \\ 2 & 43 \\ & 865 \\ 11 & 5 \\ 113 & 2075 \\ 200 & 00165 \\ \hline 328 & 70415 \end{array}$$

EXPLANATION.—Since by the decimal system numbers increase in value from right to left in a ten-fold ratio, and the decimal point separates integral from fractional orders, observe to write decimals so that the *points* fall in the same vertical line, as units of the same order will thus fall in the same column; the result of the addition is then obtained in the same manner as in simple numbers.

Decimals, like other fractions, can be neither added nor subtracted until reduced to a common denominator; but the scale in decimals being in the uniform ratio of *ten*, it is only necessary to write decimals for addition or subtraction so that the decimal points are in the same vertical line; the columns will then be of the same orders of units; in other words the decimals will be *practically* reduced to a common denominator.

The denominator of that expression containing the highest number of places is the least common denominator of the decimals; therefore the least common denominator may in all cases be determined by inspection, and decimals may be reduced to their least common denominator by simply supplying decimal ciphers until all have the same number of places.

In practice, however, this is never done, being rendered unnecessary by observing to write decimals so that the *points* stand under each other.

Rule.—*Write the decimals so that the points are in the same vertical line. Add as in whole numbers, and place the point in the sum directly below the points in the numbers added.*

EXAMPLES.

218. *1.* Find the sum of 15.4, 185.6, 106.75, 1.875, .18065.

2. What is the sum of 14.8 # + 19.65 # + 42.2 # + 75.17 #?

3. Add \$$12\frac{3}{4}$, \$$185\frac{1}{2}$, \$$195\frac{4}{5}$, \$$199\frac{1}{4}$, \$$16\frac{8}{10}$, \$$175\frac{13}{20}$, \$$98\frac{75}{100}$.

4. Add \$196.96, \$$198\frac{4}{5}$, \$$96\frac{75}{100}$, \$14.085, \$$195.12\frac{1}{2}$, $108.16\frac{1}{4}$.

5. A man cut $16\frac{1}{2}$ cords of wood the first week, 15.76 cords the second week, and $9\frac{5}{8}$ cords the third week. How many cords did he cut in all?

6. Add 1.8 ozs., 16.75 ozs., $19\frac{1}{4}$ ozs., 14.9 ozs., 1.65 ozs.

7. How many thousand feet of lumber in four piles measuring as follows: 16815, 28185, 16189, 75141 feet, respectively?

8. How many tons of coal in 5 car loads weighing respectively: 22.815 tons, 21.86 tons, 20.185 tons, 19.998 tons, 18.125 tons?

9. Add 18.195 + 17.006 + 73.0008 + 199.01 + 37.0508 + 14.1985 + 16.1463 + 27.193 + 16.175, and express the result in integers and a common fraction.

10. A lumber dealer bought 6 car loads of lumber as follows: 1 car of 16.185 M at a cost of \$178.12, 1 car of 15.998 M at a cost of \$$198.37\frac{1}{2}$, 1 car of 14.17 M at a cost of \$132.88, 1 car of 19.185 M at a cost of \$$300.96\frac{1}{2}$, 1 car of 20.145 M at a cost of \$418.50 and 1 car of 17.500 M at a cost of \$175. Find the total number of feet bought and the total cost of same.

SUBTRACTION.

219. To subtract decimals.

EXAMPLE.—Subtract .17 from .56.

OPERATION.

$$\begin{array}{r} .56 \\ .17 \\ \hline .39 \end{array}$$

EXPLANATION.—For reasons heretofore explained, place the subtrahend below the minuend, so that the decimal points shall fall in the same vertical line. Subtract as in simple numbers, and place the *point* in the remainder below the points in the terms above.

In case the number of decimal places of the subtrahend are greater than those of the minuend, consider decimal ciphers as annexed to the minuend, and subtract as before.

Mixed decimals may be treated in the same manner.

Rule.—***Write the terms in decimal order and subtract as in integers, placing the point in the remainder below the points in the other terms.***

EXAMPLES.

220. *1.* From .985 take .357.

2. Subtract .008 from .668.

3. Subtract .09 from .998.

4. From 12.684 brls. take 5.78 brls.

5. From 185.75 M feet lumber take 85.25 M feet.

6. From a purse containing $18.95 take $7.98.

7. Diminish $189.75 by $98.89½.

8. A merchant having $185.98 bought 175 # cheese at 8½¢ per lb., 75 # Rio coffee at 33¢ per lb., and 100 lbs. tea at 50¢ per lb. What amount in dollars and decimals of a dollar did he have remaining?

MULTIPLICATION.

221. To multiply decimals.

EXAMPLE.—Multiply .172 by .14.

OPERATION.

$$.172 = \tfrac{172}{1000}.$$

$$.14 = \tfrac{14}{100}.$$

$$\tfrac{172}{1000} \times \tfrac{14}{100} = \tfrac{2408}{100000}.$$

$$\tfrac{2408}{100000} = .02408.$$

EXPLANATION.—Since .172 = $\frac{172}{1000}$ and .14 = $\frac{14}{100}$, therefore $\frac{172}{1000} \times \frac{14}{100} = \frac{2408}{100000}$. The *numerator* (2408), is found by *multiplying* 172 by 14, while the *denominator* (100000), is found by *adding* to 1 (the product of 1 × 1), the *three* ciphers in the denominator of the multiplicand (1000), and the *two* ciphers in the denominator of the multiplier (100), making a total of *five ciphers* annexed to 1, which is read *hundred-thousandths.*

SECOND OPERATION.

```
  .172
   .14
 -----
   688
  172
 -----
.02408
```

EXPLANATION.—Observe that the same process is carried out in multiplying decimals, for .172 × .14 = .02408. The *numerator* of the decimal is 2408, produced by *multiplying* 172 × 14. The *denominator*, *hundred-thousandts* (100000), is produced by adding the *three* decimal places indicated as the denominator in .172 to the *two* decimal places indicated as the denominator of .14, making a total of *five* decimal places annexed to the decimal point, and read *hundred-thousandths.*

Rule.—***Multiply as in whole numbers; then, from the right of the product, point off for decimals a number of places equal to the number in both factors, prefixing ciphers if needed to obtain the required number.***

Any decimal may be multiplied by 10, 100, 1000, etc., by removing the decimal point in the multiplicand as many places to the *right* as there are ciphers in the multiplier.

EXAMPLES.

222. Multiply

1. .75 by .5.
2. .17 by .8.
3. .62 by .12.
4. .142 by .14.
5. 1.625 by .19.
6. 8.37 by 1.8.
7. 16.25 by 8.4.
8. 9.125 by .08.
9. 1.86½ by .46⅔.
10. $75.12 by 1.58.
11. $18.92 by .16⅔.
12. 7.59 by .75⅔.

13. Sold Mohawk Valley Lumber Co. for cash 185.998 M feet pine lumber at $18.87½ per M. Find amount of invoice.

14. Bot. of Merriam & Co. 5 car loads of lumber at $16.62½ per thousand feet, which inspect as follows: Car No. 1462—16.849 M; Car #853—19.854 M; Car #985—21.413 M; Car #4575—20¾ M; Car #851—21.54½ M. Find amount of invoice.

DIVISION.

223. Since division is the reverse of multiplication, and as in multiplication the decimal places of the two factors are added to determine the number of decimal places in the product, therefore in division the number of decimal places in the quotient are found by subtracting the decimal places, if any, in the divisor from those in the dividend.

224. To divide decimals.

EXAMPLE.—Divide .02408 by .172.

OPERATION.

```
.172 ) .02408 ( .14
        172
        ---
         688
         688
         ---
```

EXPLANATION.—Divide as in whole numbers; the divisor has 3 decimal places; the dividend has two more decimal places than the divisor, therefore, point off two places from the right of the quotient.

Rule.—I. *When needed, annex ciphers to the dividend to make its places equal in number to those of the divisor.*

II. *Divide as in integers, and, from the right of the quotient, point off for decimals as many places as the number of places in the dividend exceeds those in the divisor.*

Any decimal may be divided by 1 with any number of ciphers annexed, as 10, 100, 1000, 10000, by removing the *decimal point* as many places to the left as the divisor contains ciphers.

SUGGESTIONS.—1. Do not commence the division until the number of decimal places in the dividend is at least equal to the number of decimal places in the divisor. Supply any deficiency in the dividend by annexing ciphers.

2. If the divisor and dividend have the same number of decimal places, the quotient obtained, to the limit of the dividend as given, will be a whole number.

3. If the number of decimal places in the dividend be greater than the number of decimal places in the divisor, point off from the right of the quotient for decimals, a number of places equal to such excess, prefixing ciphers to the quotient if necessary.

4. If after division there be a remainder, ciphers may be annexed to it and the division continued to exactness, or to the discovery of a repetend, or to the two or three places ordinarily demanded in business computations. All such added ciphers should be considered as parts of the dividend.

EXAMPLES.

225. Find the quotients of

1. 625 ÷ 2.5.
2. 1.25 ÷ .05.
3. 14.4 ÷ 1.2.
4. 172.8 ÷ .144.
5. 25.6 ÷ 51.2.
6. 1.8 ÷ 900.
7. .008 ÷ 400.
8. 8.1 ÷ 0.9.
9. 8.5 ÷ 850.
10. 9.5 ÷ 190.
11. 15 ÷ .75.
12. 8.5 ÷ 150.

Find the sum of the quotients:

(*13*)

3 ÷ 3 = ?
3 ÷ .3 = ?
3 ÷ .03 = ?
30 ÷ .30 = ?
.3 ÷ 3 = ?
.03 ÷ 3 = ?

(*14*)

.44 ÷ .22 = ?
4.4 ÷ .022 = ?
440 ÷ 2200 = ?
.044 ÷ 220 = ?
.00044 ÷ 22000 = ?
4.4 ÷ .00022 = ?

(*15*)

75. ÷ .15 = ?
7.5 ÷ .15 = ?
.75 ÷ .015 = ?
750. ÷ .0015 = ?
.075 ÷ .015 = ?
7500 ÷ .00015 = ?

(*16*)

1 ÷ 1 = ?
1 ÷ .1 = ?
1 ÷ .01 = ?
10 ÷ .1 = ?
10 ÷ .01 = ?
.1 ÷ 1 = ?
.1 ÷ .1 = ?
.1 ÷ .01 = ?
1 ÷ .001 = ?
.1 ÷ 10 = ?

(*17*)

.22 ÷ 11 = ?
2.2 ÷ .011 = ?
220 ÷ 11000 = ?
.022 ÷ 110 = ?
.00022 ÷ 110 = ?
2.2 ÷ .000011 = ?
2200 ÷ .011 = ?
.022 ÷ 1100 = ?
.0022 ÷ 1100 = ?
220 ÷ .0022 = ?

BUSINESS EXAMPLES.

226. *1.* Find the cost of 11.6 bales of cloth, each bale containing 61⅝ yards, at $1.54 per yard.

2. A contractor received $354.06 for excavating a cellar, at 35¢ per cubic yard. How many yards of earth were removed?

3. A house cost $4500, and the lot upon which it was built cost .62 as much as the house. Find the total cost of both.

4. A grocer bought an invoice of sugar and sold .62 of it for cash; .18 of it on time, and the remainder proved to be damaged, which cost $7.50. Find the entire amount of the invoice.

5. Exchanged 26 M shingles at $3 per M; 75 doors at $2 each, and 20 windows at $1.50 each, for 35 tons coal at $5.25 per ton; 10 cords wood at $4.25 per cord, and received cash for balance. How much cash was received?

6. Sold a lumberman 381.25 pounds of butter, at $.2875 per pound; 2468 pounds of cheese at, $.115 per pound; and 2356.5 pounds of dressed beef at $.07⅞ per pound, and received pay in lumber at $23.125 per thousand feet. How many thousand feet of lumber should be received?

7. A commission merchant makes sales for the month of January aggregating $13500. Of this .48 was for cotton goods, .15 for boots and shoes, .17 for gloves, and the remainder for notions. Find the amount of the sales in each department.

8. F. N. Banks bought of C. N. Willis & Co., dealers in flour and feed, 825 brls. City Mills flour, $6.70 per brl.; 300 sacks pastry flour, $1.62½ per sack; 250 sacks bran, $1.15 per sack; 500 sacks buckwheat, 68¢ per sack; 1500 # corn meal, 2¾¢ per lb. Find the total cost.

9. In a school of 480 students, .75 of them study bookkeeping; .50 of the remainder study short hand, .25 of the remainder practice type-writing, and the remainder study the English branches. How many students in each department?

10. Find the cost of the following invoice of lumber.

654 ft.	Pine lumber,	$15.00 per M.
386 ft.	Hemlock lumber	$12.00 per M.
1486 ft.	Spruce lumber	$16.00 per M.
960 ft.	Old pine lumber	$80.00 per M.
2285 ft.	Hemlock sheathing	$18.00 per M.
4698 ft.	Spruce flooring	$20.00 per M.
98416 ft.	Box lumber	$28.00 per M.
285 ft.	Clear pine	$72.00 per M.
914 ft.	Seconds	$40.00 per M.
256 ft.	Moulding	15¢ per foot.
385 ft.	Moulding	18¾¢ per foot.
915 ft.	Black walnut	$15.00 per C.
814 ft.	Ohio cherry	$18.00 per C.

For articles bought by the M cut off three decimal places in the quantity before multiplying; by the C two decimal places.

DENOMINATE NUMBERS.

227. A **Denominate Number** is a concrete number and may be either simple or compound.

228. A **Simple Denominate Number** is a concrete unit or a collection of concrete units of but one denomination; as, 5 lbs.; 4 years.

229. A **Compound Denominate Number** is a concrete number expressed in units of different denominations; as, 5 lbs. 4 ozs.; 4 yrs. 2 mos. 5 ds.

230. **Compound Numbers** express divisions of time, and of the money, weights and measures of different countries.

MEASURES OF WEIGHT.

231. **Weight** is of three kinds, Troy, Avoirdupois and Apothecaries. The standard of weight is the Troy pound which contains 5760 grains.

TROY WEIGHT.

232. **Troy Weight** is used for weighing precious metals, and is the standard at the mint; also in philosophical experiments.

TABLE.

24 grains (grs.) = 1 pennyweight (pwt.).
20 pwts. = 1 ounce (oz.).
12 ozs. = 1 pound (lb.).

AVOIRDUPOIS WEIGHT.

233. **Avoirdupois Weight** is used for weighing all kinds of merchandise, except the precious metals. Its standard is the Troy grain, 7000 grains making 1 pound.

TABLE.

16 ounces (ozs.) = 1 pound.
25 lbs. = 1 quarter (qr.).
4 qrs. or 100 lbs. = 1 hundred weight (cwt.).
20 cwt. = 1 Ton (T.).

At the U. S. Custom house, in weighing goods upon which duties are to be paid, and in weighing coal and iron at the mines, the *long* or *gross* ton of 2240 lbs. is used.

COMPARATIVE TABLE.

The Troy pound = 5760 grs.; the ounce, 480 grs.
The Avoirdupois lb. = 7000 grs.; the ounce, 437½ grs.

Drugs and medicines are bought and sold by Avoirdupois weight only, but for the purpose of compounding medicines the 12 ounces of the Troy pound are divided into 8 drams, each dram into 3 scruples, and each scruple into 20 grains.

MEASURES OF CAPACITY.

DRY MEASURE.

234. Dry Measure is used in measuring dry articles, such as grain and produce of all kinds. The U. S. standard is the level bushel which is 18½ inches in circular measure and 8 inches deep, containing 2150.42 cu. inches. The heaped bushel contains 2747.71 cubic inches.

TABLE.

2 pints (pts.) = 1 quart (qt.).
8 qts. = 1 peck (pk.).
4 pks. = 1 bushel. (bu.).

LIQUID MEASURE.

235. Liquid Measure is used in measuring liquids. The standard is the gallon, which contains 231 cubic inches.

TABLE.

4 gills (gi.) = 1 pint (pt.).
2 pts. = 1 quart (qt.).
4 qts. = 1 gallon (gal.).
31½ gals. = 1 barrel (brl.).
2 brls. = 1 hogshead (hhd.).

The barrel and hogshead are not fixed measures, but vary for commercial purposes.

MEASURES OF EXTENSION.

236. Extension is that which has one or more of the dimensions, length, breadth and thickness; it may therefore refer to a line, a surface or a solid. The U. S. standard is the yard of 36 inches in length.

A line has one dimension, length; a surface or area has two dimensions, length and breadth; a solid has three dimensions, length, breadth, and thickness.

LINEAR MEASURE.

237. Linear or Long Measure is used in measuring lengths and distances.

TABLE.

12 inches (in.) = 1 foot (ft.).
3 ft. = 1 yard (yd.).
$5\frac{1}{2}$ yds. or $16\frac{1}{2}$ ft. = 1 rod (rd.).
320 rds. = 1 mile (mi.).

1 Mile = 320 rods, or 5280 feet, or 63360 inches.

SURVEYOR'S LINEAR MEASURE.

238. Surveyor's Linear Measure is used by land surveyors in measuring roads and boundaries of land. A Gunthers chain, 4 rods or 66 feet, is the unit of measure.

TABLE.

$7\frac{92}{100}$ inches = 1 link (l.).
25 links = 1 rod (rd.).
4 rods = 1 chain (ch.).
80 chains = 1 mile (mi.).

SURVEYOR'S SQUARE MEASURE.

239. Surveyor's Square Measure is used in measuring land by acres and sections. The unit of land measure is the acre.

TABLE.

625 square links (sq. l.) = 1 square rod (sq. rd.).
16 square rods = 1 square chain (sq. ch.).
10 square chains, or 160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).

SQUARE MEASURE.

240. Square Measure or surface measure is used in computing areas or surfaces.

TABLE.

144 square inches = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).
36 sq. mi. (6 miles square) = 1 Township (Tp.).

CUBIC MEASURE.

241. Cubic Measure is used in measuring solids or volume.

TABLE.

1728 cubic inches = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord (cd.).

A cord of wood, which contains 128 cubic feet, is 8 feet long, 4 feet high, and 4 feet thick. For each running foot on the ground there are 16 cubic feet, and hence 8 cord feet equal a cord.

Formerly a perch of masonry was 24¾ cu. ft.; but the perch of 16½ cu. ft., which is 16½ ft. long, 1 ft. high, and 1 ft. wide, is now in general use.

A cubic yard of earth is called a load.

Mechanics estimate their work on walls by the girt, and no allowance is made for windows or doors. In estimating the amount of material required, such allowances are made.

MEASURES OF MONEY.

242. Money is the measure of value, and is used as a circulating medium of exchange.

UNITED STATES MONEY.

243. United States Money is the legal currency of the United States. The unit of United States money is the gold dollar.

TABLE.

10 mills = 1 cent (¢ or ct.).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 Eagle (E.).

The mill is not a coin, and simply represents a fraction of a cent. In calculations of United States money every price is considered as cents, or as dollars and decimals of a dollar, hence the

PRACTICAL TABLE.

100 cents (¢ or ct.) = 1 dollar ($).

CANADA MONEY.

244. Canada Money is the legal currency of the Dominion of Canada, and is, like United States money, a decimal currency.

TABLE.

10 mills = 1 cent (ct.).
100 cents = 1 dollar ($).

The Silver coins are the fifty-cent piece, the twenty-five cent piece, the twenty-cent piece, the ten-cent piece and the five-cent piece. The Copper coin is the cent. There are no Canadian gold coins; those of England and the United States are a legal tender.

ENGLISH MONEY.

245. English or Sterling Money is the legal currency of Great Britain. The unit is the pound sterling.

TABLE.

4 farthings (far.) = 1 penny (d.).
12 pence = 1 shilling (s.).
20 shillings = { 1 pound (£.).
1 Sovereign (sov.).

The bullion value of a sovereign in United States money is $4.8665. When the balance of trade is in favor of the United States the balance is paid in sterling gold and, while the comparative value of a sovereign equals $5.00 of United States money, the bullion value is exactly $.1335, or 13 cents 3½ mills less than the comparative value; and this difference represents the difference between the commercial value and the exchange value of gold in the United States.

FRENCH MONEY

246. The **Legal Currency of France** is decimal, its unit being the *silver Franc.*

The French coins are as follows·

Gold	Silver	Bronze
100 francs,	5 francs,	10 centimes,
40 francs,	2 francs,	5 centimes,
20 francs,	1 franc.	2 centimes,
10 francs,		1 centime.
5 francs.		

TABLE.

10 millimes (m.) = 1 centime (ct.) = $.00193.
10 centimes = 1 decime (dc.) = $.0193.
10 decimes = 1 Franc (fr.) = $.193.

MONEY OF THE GERMAN EMPIRE.

247. The **Unit** is the *Mark* = $.2385 United States money. It is divided into 100 pfennigs (pennies). The silver *Thaler* = $.726 United States Money.

The German coins are:

Gold	Silver	Nickel
20 marks,	20 marks,	10 pfennigs,
10 marks,	1 mark,	5 pfennigs.
5 marks.	20 pfennigs.	

MEASURES OF TIME.

248. Time is the measure of duration. The unit of time is the mean solar day. The solar year is equal to 365 days, 5 hours, 48 minutes, 49.7 seconds—nearly 365¼ days. The year, however, is divided in the calendar into 365 days, except every fourth year, when 1 day is added to the month of February, when it is called a leap year. All years divisible by 4, except centennial years, are leap years, and when centennial years are divisible by 400 they are leap years.

TABLE.

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
365 days = 1 common year (yr.).
100 years = 1 century (C.).

COMMERCIAL TABLE.

30 days = 1 month. | 12 months = 1 year.

TABLE OF CALENDAR MONTHS.

1.	January	— 31 days.	*7.*	July	— 31 days.
2.	February	— 28–29 days.	*8.*	August	— 31 days.
3.	March	— 31 days.	*9.*	September	— 30 days.
4.	April	— 30 days.	*10.*	October	— 31 days.
5.	May	— 31 days.	*11.*	November	— 30 days.
6.	June	— 30 days.	*12.*	December	— 31 days.

Standard Time.—In 1883 the principal railroads of the United States and Canada adopted what is known as the "Standard Time System." This system divides the United States and Canada into four sections or time-belts, each covering 15° of longitude, 7½° of which are east and 7½° west of the governing or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt. The governing meridians are the 75th, the 90th, the 105th and the 120th west of Greenwich, and as these meridians are just 15° apart, there is a difference in time of *exactly* one hour between any one of them and the one next on the east, or the one next on the west; the standard meridian next on the east being one hour faster, and the one next on the west one hour slower. The time of the 75th meridian is called "Eastern Time." The time of the 90th meridian is known as "Central Time." The time of the 105th meridian is known as "Mountain Time." Time in the fourth belt, which is governed by the 120th meridian, and extends to the Pacific coast, is called "Pacific Time." The changes from one time-standard to another are made at the termini of roads, or at well-known points of departure, and where they are attended with the least inconvenience and danger. This system has been adopted by most of the principal cities for local use.

OTHER TABLES.

249. Paper in the stationery trade is sold by the following

TABLE.

24 sheets .... = 1 quire.
20 quires .... = 1 ream.
2 reams .... = 1 bundle.
5 bundles .... = 1 bale.

A bale contains 200 quires, or 4800 sheets.

TABLE FOR COUNTING.

12 units = 1 dozen.
20 units = 1 score.
12 dozen = 1 gross.
12 gross = 1 great gross.

REDUCTION.

250. To reduce denominate numbers from higher to lower denominations.

Example.—Change 1 bushel, 1 peck, 1 pint to pints.

OPERATION.

1 bu. 1 pk. 0 qts. 1 pt.
4
4 pecks
1 peck
5 pecks
8
40 quarts
2
80 pints
1 pint
81 pints.

Explanation.—Since 1 bushel = 4 pecks, 1 bushel and 1 peck = 5 pecks. Since 1 peck = 8 quarts, 5 pecks = 5 × 8 quarts = 40 quarts. Since 1 quart = 2 pints, 40 quarts = 40 × 2 pints = 80 pints, and 80 pints + 1 pint = 81 pints; therefore, 1 bushel, 1 peck, 1 pint = 81 pints.

Rule.—*Multiply the units of the highest denomination given by that number which will reduce it to the denomination next lower, and add the units, if any, of that denomination. Continue in this manner until the required denomination is reached.*

EXAMPLES.

251. Reduce to lower denominations:

1. 1 lb., 8 ozs., 1 pwt. T. to grs.
2. 8 lbs., 14 ozs. Av. to ozs.
3. 1 bu., 1 pk. to quarts.
4. 1 brl., 5 gals. to gallons.
5. 18 gals., 2 qts., 1 gi. to gills.
6. 1 mi., 5 ft. to feet.
7. 1 A., 10 sq. rds. to sq. feet.
8. 10 long tons to pounds.
9. 1 cu. yd., 5 cu. ft. to cu. in.
10. 1 common yr., 50 da. to hrs.

11. Change $\frac{23}{64}$ bu. to integers and to units of lower denominations.

$\frac{23}{64}$ bu. = $\frac{23}{64}$ of 4 pks. = $\frac{92}{64}$ = $1\frac{7}{16}$ pks. $\frac{7}{16}$ pk. = $\frac{7}{16}$ of 8 qts. = $\frac{56}{16}$ qts. = $3\frac{1}{2}$ qts. $\frac{1}{2}$ qt. = $\frac{1}{2}$ of 2 pts. = 1 pt., therefore $\frac{23}{64}$ bu. = 1 pk. 3 qts. 1 pt. Reduce now to lower denominations according to rule.

12. Change .4 bushel to pecks.

.4 bushel = .4 of 4 pecks = 1.6 pecks.

13. Change 1 cord, 14 cu. feet to cu. feet.

14. Change £1, 10s. 8d. 2 far. to farthings.

15. Change $\frac{1}{90}$ bu. 1 pt. to pints.

$\frac{1}{90}$ bu. = $\frac{1}{90}$ of 4 pks. = $\frac{4}{90}$ = $\frac{2}{45}$ pecks. $\frac{2}{45}$ pk. = $\frac{2}{45}$ of 8 qts. = $\frac{16}{45}$ qts. $\frac{16}{45}$ qts. = $\frac{16}{45}$ of 2 pts. = $\frac{32}{45}$ pts., therefore $\frac{1}{90}$ bu. 1 pt. = $1\frac{32}{45}$ pints.

16. Change 5 lb. Av. to Troy grains.

17. Change $\frac{19}{64}$ bu. to integers.

18. Change .3125 gals. to lower denominations.

Multiply successively by 4 and 2 as per Art. 250.

19. Change 1 mi. 2 ch. 3 rods, 20 links to links.

20. Change 1 lb. Troy and 4 ozs. Av. to grains.

252. To reduce denominate numbers from lower to higher denominations.

EXAMPLE.—Change 67 pints to higher denominations.

OPERATION.

2) 67
8) 33 qts. + 1 pt.
4) 4 pks. + 1 qt.
67 pts. = 1 bu. 1 qt. 1 pt.

EXPLANATION.—Since 2 pints = 1 quart, 67 pints = 33 quarts and 1 pint remaining. Since 8 quarts = 1 peck, 33 quarts = 4 pecks and 1 quart remaining. Since 1 bushel = 4 pecks, the 4 pecks = 1 bushel; and the 67 pints therefore equal 1 bushel, 1 quart, 1 pint.

Rule.—I. *Divide the given number by the number of units of the same denomination required to make one of the next higher, and reserve the remainder, if any.*

II. *Proceed in like manner with the quotient, and so continue until the whole is reduced to the required denomination. The last quotient, with the several remainders, will be the required answer.*

EXAMPLES.

253. Change to units of higher denominations:

1. 129 pints. liquid measure.
2. 7000 grs. to Troy integers.
3. 185 gills.
4. 400 cu. feet to cords.
5. 963 far. sterling.
6. 17852 mills U. S. money.
7. 980 sq. rods.
8. 168 ozs. Avoirdupois.
9. 9544 sq. inches.
10. 17854 in. Surveyor's linear.

11. Change $3\frac{1}{2}$ pints to quarts.

As there are 2 pints in 1 quart, there will be as many quarts as 2 is contained times in $3\frac{1}{2}$ ($\frac{7}{2} \div 2 = \frac{7}{4} = 1\frac{3}{4}$), or $1\frac{3}{4}$ quarts.

12. What part of a pound Troy are 8 ounces Troy?

$8 \div 12 = \frac{2}{3}$ lb.; or, 8 ozs. $= \frac{8}{12}$ of 12 ozs. $= \frac{2}{3}$ lb.

13. Change $\frac{5}{8}$ ounce Avoirdupois to the decimal of a pound.

$\frac{5}{8}$ oz. $= 5 \div 8 = .625 \div 16 = .0390625$ lbs. $= .039\frac{1}{16}$ lbs.

14. Change $\frac{1}{2}$ pint and .5 quart to the decimal of a peck.

15. Change 1450 gallons to barrels of 40 gallons each.

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

254. To add denominate numbers.

EXAMPLE.—Add 1 bu., 2 pks., 1 pt.; 1 bu., 1 qt.; 1 bu., 3 pks., 7 qts., 1 pt.

OPERATION.

bu.	pk.	qt.	pt.	
1	2	0	1	
1	0	1	0	
1	3	7	1	
3	5	8	2	= first sum.
4	2	1	0	= equivalents.

EXPLANATION.—Add each denomination separately, and the simple denominations are 3 bushels, 5 pecks, 8 quarts and 2 pints. Changing to equivalants, the 2 pints = 1 quart, which added to the 8 quarts = 9 quarts; 9 quarts = 1 peck and 1 quart remaining. Write the 1 quart remaining under the column of quarts and add the 1 peck to the 5 pecks. 6 pecks = 1 bushel and 2 pecks. Write the 2 pecks under the column of pecks, and add the 1 bushel to the 3 bushels above. Therefotre, he equivalent of 3 bushel, 5 pecks, 8 quarts and 2 pints = 4 bushel, 2 pecks, 1 quart.

In practice omit the first sum and write only the equivalents.

Rule.—I. *Write the numbers so that units of the same name shall stand in the same column.*

II. *Begin at the right, add each column, and reduce the sum to the next higher denomination; write the remainder, if any, and add the quotient with the next column.*

EXAMPLES.

255. Add

1. 4 bu., 1 pk., 1 pt.; 3 pks., 1 qt., 1 pt.; 1 bu., 4 qt., 1pt.
2. 1 lb., 8 ozs., 10 grs.; 4 lbs., 5 ozs., 8 pwts., 4 grs.
3. 5 yrs., 8 mos., 8 ds.; 9 mos., 10 ds., 18 hrs., 6 mins., 20 sec.
4. 5 cords, 3 cord feet; 1 cord, 28 cu. feet.
5. 1 mi., 85 rds., 5 yds.; 3 mi., 17 rds., 4 yds., 16 ft., 9 in.; 4 mi., 17 rds., 10 ft., 10 in.
6. 10 T., 8 cwt., 3 qrs., 12 lbs.; 5 T., 2 cwt., 2 qrs., 10 lbs.
7. 5 A., 110 sq. rds., 5 sq. ft., 28 sq. in.; 1 A., 80 sq. rds., 3 sq. ft., 12 sq. in.
8. £4, 8s., 5d., 4 far.; £2, 3s., 4d., 1 far.; £6, 8d., 3 far.
9. 5 brls., 8 gals., 2 qts., 1 pt.; 3 brls., 5 gals., 1 qt., 1 pt.
10. $5, 8d., 7¢, 5 m.; $16, 14¢, 8 m.; $12, 75¢, 1 m; $4, 9d, 5¢, 5 m.; $19, 8d., 6¢, 5 m.

256. To subtract denominate numbers.

Example.—From 1 bu., 1 pk., 5 qts., take 3 pks., 4 qts., 1 pt.

OPERATION.

bu.	pk.	qt.	pt.
1	1	5	0 =
	5	4	2
	3	4	1
	2	0	1

Explanation.—Since the units of some of the denominations of the subtrahend are greater than the corresponding denominations of the minuend, change a unit of a higher denomination as in simple subtraction; therefore, 1 bushel, 1 peck, 5 quarts = 5 pecks, 4 quarts and 2 pints, and subtracting 3 pecks, 4 quarts, 1 pint, there are remaining 2 pecks, 1 pint; or reducing the denominations of each to pints, 90 pints less 57 pints = 33 pints, and 33 pints = 2 pecks, 1 pint.

Rule.—*Write the numbers to be subtracted with units of the same denomination under each other. In case any subtrahend term be greater than the corresponding minuend term, change or borrow 1 as in simple subtraction, and reduce it to the denomination required.*

EXAMPLES.

257. Find the remainders.

1. 18 lbs. T. — 8 ozs., 5 grs.
2. 16 bu., 2 qts. — 1 qt., 1 pt.
3. 5 cords — 118 cu. feet.
4. 1 hhd. — 1 gal., 4 gills.
5. £14, 3 far. — 1d. 2 far.
6. 6 mi., 14 rds. — 110 rds., 8 ft.
7. 1 A., 16 sq. rds. — 120 sq. rd.
8. 18 T. — 1 T., 5 cwt., 3 qrs.
9. 10 ds. — 1 da., 5 hrs., 8 min.
10. $25, 6d., 4¢ — $8, 5¢, 8 m.

258. **To multiply denominate numbers.**

EXAMPLE.—Multiply 1 bu., 2 pks., 2 qts., 1 pt., by 7.

OPERATION.

$$\begin{array}{rrrrl} \text{bu.} & \text{pk.} & \text{qts.} & \text{pt.} & \\ 1 & 2 & 2 & 1 & \\ & & & 7 & \\ \hline 7 & 14 & 14 & 7 & = \\ 11 & 0 & 1 & 1 & \end{array}$$

EXPLANATION.—Multiplying each denomination separately by 7 gives 7 bushels, 14 pecks, 14 quarts, 7 pints, which changed to units of higher denominations = 11 bushels, 1 quart, 1 pint.

Rule.—*Multiply each denomination separately and change the partial products to units of higher denominations.*

259. Multiply

1. 8 bu., 2 pks., 1 pt., by 9.
2. 1 mi., 110 rds., 8 in., by 6.
3. 1 brl., 4 gals., 1 qt., by 8.
4. 1 A., 14 rds., by 16.
5. 15 cords, 8 cord feet, by 9.
6. £3, 8s. 5d. 2 far., by 16.
7. 5 lbs. T., 8 ozs., 1 gr., by 24.
8. 3 T., 4 cwt., 8 lbs., by 16.
9. 3 yrs., 13 min., 18 sec., by 45.
10. $18, 75¢, 8 m., by 10.

260. **To divide denominate numbers.**

EXAMPLE.—Divide 5 bu., 1 pk., 3 qts., by 4.

OPERATION.

$$\begin{array}{rrrrr} & \text{bu.} & \text{pks.} & \text{qts.} & \text{pts.} \\ 4) & 5 & 1 & 3 & 0 \\ \hline & 1 & 1 & 2 & 1\frac{1}{2} \end{array}$$

EXPLANATION.—Dividing 5 bushels by 4 gives a quotient of 1 bushel and an undivided remainder of 1 bushel; reduce this remainder to pecks (4) and add to the 1 peck of the dividend, obtaining 5 pecks, which divided by 4 gives 1 peck and an undivided remainder of 1 peck. Reduce the 1 peck remaining to quarts (8), and adding to the 3 quarts of the dividend gives 11 quarts, which divided by 4 gives 2 quarts and an undivided remainder of 3 quarts. 3 quarts = 6 pints, which divided by 4 gives 1½ pints.

Rule.—*Divide as in simple numbers, and when necessary reduce remainders to lower denominations.*

Should the highest dividend order not contain the divisor, reduce its units to the order next lower, and so proceed to the end.

The above and like divisions may be accomplished by the reduction of the denominate expressions to the lowest order in its scale, then effecting the division and afterwards reducing the quotient to higher denominations.

EXAMPLES.

261. Divide

1. 25 A., 4 rds., by 8.
2. 95 mi., 17 rds., by 16.
3. 24 bu., 3 pks., by 9.
4. 25 cords, 8 cu. ft. by 12.
5. 10 T., 8 cwt., by 16.
6. 14 yrs., 4 ds., 10 hrs., by 8.
7. 40 lbs. T., 8 ozs., by 12.
8. £18, 10s. 4d., by 5.
9. 75 brls., 20 qts., by 6.
10. 83 bu., 3 pks., by 14.

MISCELLANEOUS MEASUREMENTS.

262. A **Plane Figure** is a plane bounded by a line or lines; the plane being a surface in which any two points being taken, the straight line connecting them will lie wholly in the surface.

263. The **Area** of a figure is the quantity of surface it contains.

To find the area of any plane figure, the opposite sides of which are equal and parallel. RULE.—*Multiply the base by the perpendicular hight.*

Most of the rules here given depend upon the principles of Geometry for explanation, but are given for convenience in business requirements.

To find the area of a plane figure, whose opposite sides are parallel but of unequal length. RULE.—*Obtain the average length, aad multiply by the perpendicular hight.*

264. A **Triangle** is a plane figure bounded by three straight lines.

To find the area of a triangle, the base and hight being given. RULE.—*Multiply the base by one-half the hight.*

265. A **Circle** is a plane figure bounded by a curved line, every part of which is equally distant from the center.

266. The **Circumference of a Circle** is the curved line bounding it.

To find the circumference of a circle, the diameter being given. RULE.—*Multiply the diameter by 3.1416.*

267. The **Diameter of a Circle** is a straight line passing through the center and terminating in the circumference.

To find the diameter of a circle, the circumference being given. RULE.—*Divide the circumference by 3.1416.*

To find the area of a circle, the circumference and diameter being given. RULE.—*Multiply the circumference by the diameter, and divide the product by 4.*

To find the side of a square equal in area to a given circle. RULE.—*Multiply the circumference by .2821.*

268. The **Radius of a Circle** is a straight line passing from the center to the circumference. It is one-half of the diameter.

269. A **Cylinder** is a circular body of uniform diameter, the ends of which are parallel circles.

To find the surface or area of a cylinder. RULE.—*Multiply the circumference by the hight.*

To find the contents of a cylinder. RULE.—*Multiply the area of the base by the hight.*

To guage or measure the capacity of a cask. RULE.—*Multiply the square of the mean diameter in inches by the length in inches, and this product by .0034; the result will be the capacity in gallons.*

270. A **Pyramid** is a solid, the base of which has three or more equal lines, and the sides of which terminate in a point called a vertex.

271. A **Cone** is a solid having a circular base, its convex surface terminating in a point called a vertex.

To find the surface of a regular pyramid or cone. RULE.—*Multiply the perimeter or circumference of the base, by one-half the slant hight.*

To find the contents of a pyramid or cone. RULE.—*Multiply the area of the base by one-third the perpendicular hight.*

272. A **Sphere** is a solid bounded by a curved surface, all points of which are equally distant from the center.

273. The **Diameter** of a sphere is a straight line drawn through its center, terminating each way at the surface.

To find the surface of a sphere. RULE.—*Multiply the square of its diameter by 3.1416.*

To find the volume of a sphere. RULE.—*Multiply the cube of its diameter by .5236.*

EXAMPLES.

274. *1.* How many square feet in the gable end of a house 24 ft. wide and 6 ft., 6 in. high?

2. Find the number of square yards in a triangular sail, the sides of which are 30 ft., 40 ft., and 50 ft. respectively.

3. How many acres in a rectangular field 108 rods long and 48 rods wide?

4. A farm stretches across an entire section, being 200 rods wide on the west line and 160 rods wide on the east line. How many acres in the farm? *

5. How many feet of fence will inclose a circular pond 82.5 ft. in diameter?

6. What is the diameter of a circle, the circumference of which is 90 rods?

7. What will be the cost of a sheet-iron smoke-stack 40 ft. high and 2 ft. in diameter, at 15¢ per square foot?

8. Find the capacity in gallons of a tank 14 ft. deep and 18 ft. in diameter?

9. A pyramid has a triangular base 3 ft. on each side, and a slant hight of 10 ft. Find the number of square feet in its surface.

10. A tent is in the form of a cone; if its slant hight is 16 ft. and its base circumference 30 ft., how many square yards of duck were used in making it?

11. How many square inches of leather will cover a foot ball 8 in. in diameter?

12. How many cubic feet in the contents of a globe 4 ft. in diameter?

13. A cask 28 in. at each end, and 34 in. at the bilge, is 3 ft. long. How many gallons of water will it hold?

MISCELLANEOUS EXAMPLES.

275. *1.* What is the value in English money of $1850.75 United States gold coin?

2. What is the value in United States money of £16, 8s. 5d. 3 far. English money?

3. If a steamship requires 1 yr., 9 mo., 14 ds. to circumnavigate the globe, what time would be required in 8 trips of equal time?

4. How much coal in 5 car loads weighing respectively 18 T., 4 cwt., 80 lbs.; 16 T., 10 cwt., 50 lbs.; 19 T., 5 cwt., 90 lbs.; 18 T., 18 cwt.; 20 T., 12 cwt., 18 lbs.

5. How many lbs., ozs., pwts. and grs. in 10 cwt., 16 lbs., 11 ozs. Avoirdupois?

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* A section of land is 1 mile wide.

6. How many barrels of water will a cistern hold 5 ft., 4 in. long, 3 ft., 10 in. wide and 8½ ft. deep?

7. How many rods of fence will be required to enclose a field 840 feet long by 650 feet wide?

8. How many square feet of sheet lead will be required to line a tank 7 ft. in diameter and 12 ft. deep?

9. How many francs are equal to $22000 U. S. gold coin?

10. If a furnace consumes 1 cwt., 8 lbs. coal per day, how much will be required to run the furnace 150 days?

11. How many bushels of grain, stricken measure, will a bin contain 25 ft., 8 in. long, 14 feet deep and 6½ feet wide?

12. What shall be the length of a shed 8½ high and 8½ feet wide to contain 250 cords of wood?

13. How many pounds and dec. of a pound T. in 5 lbs. Av.?

14. A druggist bought 250 lbs. epsom salts at 75¢ per lb. Av. and sold them at 15¢ per oz. T. How much did he make?

15. At $1.75 per rod, what will be the cost to fence a field 180 rods long by 32 rods wide?

16. Find the cost of 10 T., 8 cwt., 90 lbs. coal at $4.87½ per Ton.

17. The Lehigh Valley Coal Co. have on hand Sept. 1, 1891, long tons as follows of coal, 875 T., 11 cwt. How many short tons and what is the value at $4.25 per short ton?

18. A German immigrant has 1500 thalers and 800 marks to exchange for United States money. How many dollars should be paid him?

19. At $85.25 per A, what is the cost of 5 fields, each containing 10 A, 14 sq. rds. land? What will be the total cost if 1 T., 8 cwt. fertilizer be used on each field at a cost of $5.50 per ton?

20. If a car carrying 20 tons of freight is with its couplings 42 ft. long, what would be the length of a train carrying Vanderbilt's two hundred millions of dollars, if it is all in standard silver dollars, and any fractional part of a car load is rejected?

A standard silver dollar weighs 412.5 grs.

SPECIAL APPLICATIONS.

276. Special Applications.—The essential elements of every business transaction involving purchases and sales are, cost, price and quality. Special Applications as here treated embrace the use, in the solution of problems, of any or all explanations heretofore given, and also of such contracted methods as may be employed in dealing with aliquot parts.

ALIQUOT PARTS.

277. The **Aliquot Parts** of a number are the even parts of that number; as, the aliquot parts of 100 are 50, 33⅓, 25, etc.

The aliquot parts of $1.00 are useful in contracted computations where quantity and price are given to find the cost.

The aliquot parts of years, months and days are used in contracted methods in interest computations.

The component factors of a number must be integral, while the aliquot parts of a number may be either integral or mixed.

ALIQUOT PARTS AND PARTS OF PARTS.

Parts of $1.00.	Parts of 50¢.	Of 25¢.	More or Less than $1.00.
50¢ = ½			112½ = ⅛ more
33⅓¢ = ⅓			125¢ = ¼ more
25¢ = ¼	or ½ of		133⅓ = ⅓ more
20¢ = ⅕			110¢ = 1/10 more
16⅔¢ = ⅙	or ⅓ of		90¢ = 1/10 less
12½¢ = ⅛	or ¼ of	or ½ of	87½¢ = ⅛ less
10¢ = 1/10	or ⅕ of		80¢ = ⅕ less
8⅓¢ = 1/12	or ⅙ of	or ⅓ of	83⅓¢ = ⅙ less
6¼¢ = 1/16	or ⅛ of	or ¼ of	75¢ = ¼ less
5¢ = 1/20	or 1/10 of	or ⅕ of	66⅔¢ = ⅓ less
3⅓¢ = 1/30	or 1/15 of		62½¢ = ⅜ less
2½¢ = 1/40	or 1/20 of	or 1/10 of	40% = ⅗ less
1⅔¢ = 1/60	or 1/30 of	or 1/15 of	37½ = ⅝ less

Based upon the above table short methods of computation may be used in reference to the quantity by using 100 units of the quantity as the basis.

General Rules.—1. *Find the cost of the total quantity by multiplying the given quantity by $1.00; then take such a part of the product thus obtained, more or less, as the given price is a part, more or less, of $1.00.*

2. *Find the cost of 100 units of the quantity (in weight, measure or volume, as the case may be) by multiplying these 100 units by the given price; then, take such a part of the product thus obtained, more or less, as the given quantity is more or less than 100.*

278. To find the cost when the price is an aliquot part of a dollar.

EXAMPLE 1.—Find the cost of 672 gals. N. O. Molasses at 33⅓¢ per gallon.

OPERATION.

672 × $1.00 = $672.
⅓ of $672. = $224.

EXPLANATION.—The cost of 672 gals. at $1.00 will be $672. Since 33⅓¢ = ⅓ of $1.00, the cost of 672 gals., at 33⅓¢, will equal ⅓ of $672, or $224.

EXAMPLE 2.—Find the total cost of 150 yds. Broad Cloth at $1.25 per yard; 16 yds. Jersey Cloth at 87½¢ per yard; 90 yds. Cambric at 16⅔¢ per yard.

OPERATION.

150 × $1.00 = $150.00
¼ of 150 = 37.50 = $187.50
16 × $1.00 = $16.00
⅛ of $16 = 2.00 = $14.00
90 × $1.00 = $90.00
⅙ of $90 = = $15.00
Total, $216.50

EXPLANATION.—The cost of 150 yds. at $1.25 = 1¼ × the cost of 150 yds. at $1.00, or $187.50; the cost of 16 yds. at 87½¢ = ⅛ less than the cost of 16 yds. at $1.00 per yd., or $14.00; the cost of 90 yds. at 16⅔¢ = ⅙ the cost of 90 yds. at $1.00 per yd., or $15.00.

Rule.—*Take such a part of the quantity, considered as dollars, as the given price is a part, more or less, of $1.00.*

EXAMPLES.

279. *1.* Find the total cost of

350 #	Tea,	50¢ per lb.
870 #	Coffee,	33⅓¢ per lb.
125 #	Raisins,	25¢ per lb.
85 #	Raisins,	16⅔¢ per lb.
150 #	Rice,	10¢ per lb.
75 #	Starch,	12½¢ per lb.
140 #	Soda,	6¼¢ per lb.
124 #	Meal,	3⅓¢ per lb.

2. Find the total cost of

840 yds.	Jeans,	20¢ per yd.
175 yds.	N.Y.Mills,	6¼¢ per yd.
90 yds.	Silesia,	16⅔¢ per yd.
72 yds.	Drilling,	8⅓¢ per yd.
1040 yds.	Prints,	5¢ per yd.
950 yds.	Ticking,	10¢ per yd.
140 yds.	Linings,	25¢ per yd.
350 yds.	Plaids,	33⅓¢ per yd.

3. Find the total cost of

44 yds. Suiting,	$2.25	per yd.
75 yds. Lace,	$1.33⅓	per yd.
60 yds. Lace,	87½¢	per yd.
225 yds. Mohair,	75¢	per yd.
140 yds. Alpaca,	66⅔¢	per yd.
850 yds. Plaids,	62½¢	per yd.

4. Find the total cost of

286 lbs. Tea,	50¢ per lb.
1152 yds. Linen,	33⅓¢ per yd.
527 lbs. Lard,	12½¢ per lb.
455 gals. Molasses,	25¢ per gal.
1751 doz. Eggs,	16⅔¢ per doz.
2133 lbs. Pork,	6¼¢ per lb.

5. Brown, Mead & Co.,

Bot. of The Co-operative Grocery Co.:

175 # Ham,	16⅔¢	per lb.
140 # London Layers,	18¾¢	per lb.
150 # Japan Tea,	40¢	per lb.
112 gals. Maple Molasses,	90¢	per gal.
54 sacks Flour,	$1.12½	per sack.
75 sacks Flour,	$1.62½	per sack.
62 bu. Potatoes,	$1.12½	per bu.

Find total cost.

6. Elias W. Whitney,

Bot. of Benj. N. Scott, Sons & Co.:

16 doz. cans Corn,	$1.25	per doz.
152 # English Breakfast Tea,	37½¢	per lb.
155 doz. cans Peaches,	$1.33⅓	per doz.
56 gals. Golden Drip,	83⅓¢	per gal.
850 # Bran,	$1.10	per C.
190 doz. cans Tomatoes,	80¢	per doz.
65 doz. cans Cherries,	$1.16⅔	per doz.

Find total cost.

280. To find the cost when the price is given and the quantity is a multiple or an aliquot of 100.

EXAMPLE.—Find the total cost of 125 yds. Cambric, 54¢; 150 yds. Prints, 6¾¢; 175 yds. Sheeting, 18¢.

OPERATION.

100 yds. @ 54¢ =	$54.00	
25 yds. = ¼ =	13.50	$67.50
100 yds. @ 6¾¢ =	$6.75	
50 yds. = ½ =	3.38	$10.13
100 yds. @ 18¢ =	$18.00	
50 yds. = ½ =	9.00	
25 yds. = ½ (of 50)	4.50	$31.50
		$109.13

EXPLANATION. — The cost of 100 yds. at 54¢ = $54.00, hence the cost of 125 yds. will be ¼ more, or $67.50; the cost of 100 yds. at 6¾¢ = $6.75, and hence 150 yds. will cost ½ more, or $10.13; the cost of 100 yds. at 18¢ = $18.00, hence the cost of 175 yds. will be ½ and ¼ more, or $31.50.

Rule.—*Take such a part of the cost of 100 units of the given quantity at the given price, as the given quantity is more or less than 100 or times 100.*

EXAMPLES.

281. *1.* Find the cost of

200 # Tea, 38¢ per lb.
150 # Coffee, 33⅓¢ per lb.
175 # Cod Fish, 8½¢ per lb.
125 cans Cocoa, 18¢ per can.
700 brls. Flour, $5.70 per brl.

2. Find the cost of

800 yds. Cotton, 13¢ per yd.
600 yds. Cambric, 22¢ per yd.
400 yds. Drilling, 23¾¢ per yd.
150 yds. Alpaca, 78⅓¢ per yd.
500 yds. Silk, $1.50 per yd.

3. Find the cost of

200 lbs. Tea, 37½¢ per lb.
700 lbs. Chocolate, 51¼¢ per lb.
150 lbs. Raisins, 14¾¢ per lb.
250 lbs. Coffee, 21⅗¢ per lb.
1000 sks. Flour, $1.12½ per sk.
750 gals. Molasses, 81⅗¢ per gal.
1250 gals. Syrup, $1.62½ per gal.

4. Find the cost of

300 yds. Woolen, 41¾¢ per yd.
3000 yds. Gingham, 12⅓¢ per yd.
2500 yds. Mohair, $1.10 per yd.
25 yds. Brilliant. $1.85 per yd.
150 yds. Satteen, 33⅓¢ per yd.
75 yds. Diagonals, $1.15 per yd.
125 yds. Brocade, $1.25 per yd.

282. **Masonry** is computed in cubic feet, in perches, and sometimes in square feet and square yards.

A perch of masonry is 16½ cubic feet.

Mechanics estimate their work on walls on the *outside*, and, usually, no allowance is made for doors, windows and corners. In estimating the amount of material required, such allowances are made.

283. To find the number of perches in a wall.

EXAMPLE.—How many perches of stone in a stone wall 60 feet long, 4 feet high and 1½ feet thick?

OPERATION.

$$60 \text{ ft.} \times 4 \times 1\tfrac{1}{2} = 360 \text{ cu. ft.}$$
$$360 \text{ cu. ft.} \div 16\tfrac{1}{2} = 21\tfrac{9}{11} \text{ perches.}$$

EXPLANATION.—Since there are 16½ cubic feet in 1 perch of stone, there are as many perches as 16½ cubic feet are contained times in the total number of cubic feet of the wall; and $360 \div 16\frac{1}{2} = 21\frac{9}{11}$.

Rule.—*Divide the number of cubic feet in the wall by 16½.*

284. Brickwork is generally estimated by the *thousand bricks,* but sometimes by cubic feet.

North River bricks are in size 8 in. long, 3½ in. wide and 2¼ in. thick. Maine bricks, 7½ in. × 3⅜ in. × 2⅜ in.; Philadelphia and Baltimore, 8¼ in. × 4⅛ in. × 2⅜ in.; Milwaukee, 8½ in. × 4⅛ × 2⅜; common or ordinary bricks, 8 in. × 4 in. × 2 in.

285. To find the number of bricks in a cubic foot of wall.

EXAMPLE.—How many North River bricks in a cubic foot of brick work if the bricks are laid in mortar ¼ of an inch thick.

OPERATION.

8¼ in. × 3¾ × 2½ = $77\frac{11}{32}$ cu. in.
1728 cu. in. ÷ $77\frac{11}{32}$ = 22⅓ bricks.

EXPLANATION.—By adding ¼ inch to each dimension 8 inches + ¼ inch = length, 3½ inches + ¼ inch = width, and 2¼ inches + ¼ inch = thickness; 8¼ inches × 3¾ × 2½ = $77\frac{11}{32}$ cubic inches in 1 brick and mortar. Since 1 cubic foot contains 1728 cubic inches, there will be required as many bricks as $77\frac{11}{32}$ is contained times in 1728, or nearly 22⅓ bricks.

Rule.—I. *Multiply together each dimension of the brick increased by the thickness of the mortar, and the product will be the volume of 1 brick and its mortar.*

II. *Divide 1728, the number of cubic inches in 1 foot, by the volume of 1 brick and its mortar, and the quotient will be the number of bricks in a cubic foot of brick wall.*

286. To find the volume of a solid.

EXAMPLE.—How many cubic feet of sod in a pile of sod 18 feet long, 10 feet high and 15 feet wide?

OPERATION.

18 ft. × 15 × 1 = 270 cu. ft.
270 cu. ft. × 10 = 2700 cu. ft.

EXPLANATION.—By multiplying together the length, 18 feet, by the width, 15 feet, and this product by 1 foot gives the number of cubic feet in the ground layer 1 foot high. Multiplying this product, 270 cubic feet, by the number of layers, 10, gives 2700 cubic feet in the whole pile.

Rule.—*Multiply together the length, breadth and thickness, expressing each in the same denomination, and the product will be the number of cubic units of that denomination.*

287. To find the gallon measure of a vessel or other space.

Example.—Find the contents in gallons of a cistern 5 feet square and 6 feet deep.

OPERATION.

5 ft. × 5 × 6 = 150 cu. ft.
150 cu. ft. × 1728 = 259200 cu.in.
259200 cu. in. ÷ 231 = 1122$\frac{6}{77}$ gals.

Explanation. — Since 1 liquid gallon contains 231 cubic inches, there are as many gallons in the cistern as 231 cubic inches are contained times in the volume of the cistern expressed in cubic inches (5 ft. × 5 × 6 × 1728 ÷ 231), or 1122$\frac{6}{77}$ gals.

Rule.—*Divide the contents expressed in cubic inches by 231 for liquid gallons, and by 268.8 for dry gallons.*

Six dry gallons are equal to nearly seven liquid gallons.

288. To find the bushel contents of a bin or elevator.

Example.—How many bushels of grain will a bin contain 62 in. × 50 in. × 30 in.?

OPERATION.

62 in. × 50 × 30 = 93000 cu. in.
93000 cu. in. ÷ 2150.42 = 43$\frac{1}{4}$ bu.

Explanation.—As 1 bushel contains 2150$\frac{42}{100}$ cubic inches, there will be as many bushels as 2150.42 cubic inches are contained times in the volume of the bin expressed in cubic inches. 62 in. × 50 × 30 = 93000 cu. in., and 93000 cu. in. ÷ 2150.42 = 43$\frac{1}{4}$ bu. nearly.

Rule.—*Divide the volume in cubic inches by 2150.42 for stricken measure, and by 2747.71 for heaped measure.*

If the volume is expressed in feet deduct $\frac{1}{5}$ for stricken measure.

If the crib flare take the mean width.

Grain, seeds, and small fruits are sold by stricken measure.

Corn in the ear, potatoes, coal, large fruits, coarse vegetables and other bulky products are sold by heaped measure.

About 5 stricken measures equal 4 heaped measures.

289. To find the weight of hay in a stack or load.

Owing to pressure which varies according to the hight, time of settling and obstruction in packing, the accepted measurements of hay are of three kinds:

1st. To find the weight of hay in load or shed loft, unpressed. Rule.—*Divide the volume in cubic feet by 540 cubic feet, and the product will be in tons.*

2nd. To find the weight in common hay barn, or small stack. Rule.—*Allow 405 cubic feet for a ton, and divide as in rule 1st.*

3rd. To find the weight in mow bases in barns, compressed with grain, and in butts of stacks of timothy hay. RULE.—*Allow 32¼ cubic feet for a ton, and divide as in rule 1st.*

290. To find the number of yards of carpeting required to cover a floor.

EXAMPLE.—How many yards of carpeting, ¾ of a yard wide, will be required to cover the floor of a room 18 feet wide by 32 feet long, the strips to run the longer way, the waste being 12 inches on each strip?

OPERATION.

$32 \text{ ft.} \div 3 \text{ ft.} = 10\frac{2}{3} \text{ yds.}$

$\text{Wastage,} \quad \underline{\tfrac{1}{3} \text{ yd.}}$

$\text{Length of strips} = 11 \text{ yds.}$

$18 \text{ ft.} \div 3 \text{ ft.} = 6 \text{ strips, } \frac{4}{4} \text{ wide.}$

$6 \text{ strips} + \frac{1}{3} (\frac{1}{3} \text{ of } 6) = 8 \text{ strips, } \frac{3}{4} \text{ wide.}$

$8 \times 11 \text{ yds.} = 88 \text{ yds.}$

EXPLANATION.—Since the strips are to run the longer way of the room, each strip in yards will be ⅓ of 32 feet + 1 foot, or 11 yards. Since the width of each strip (¾ yard) is 2¼ feet, there will be required as many strips as 2¼ feet are contained times in 18 feet, the width of the room; and $18 \div 2\frac{1}{4} = \frac{72}{4} \div \frac{9}{4} = 8$ *times* or 8 strips, and 8 times 11 yards = 88 yards.

Rule.—I. *To get the length of each strip, divide the dimension of the room the way the strips are to lay by 3, and add the wastage, if any; the quotient will be the length in yards of each strip.*

II. *To get the number of strips required, divide the other dimension by the width, in feet, of the carpet; the quotient will be the number of strips.*

III. *Multiply the length of each strip by the number of strips and the product will be the number of yards required for the room.*

A yard of carpeting refers to the running measurement in the roll or strip, regardless of width. To get the number of strips 1 yd, wide, divide the dimension in feet by 3 and the quotient will be the number of strips 1 yard wide. For ¾ yd. carpeting add ⅓ to the number for 4/4; for ⅔ yds. wide add ½, etc.

When the width of the room is not exactly divisible by the width of the carpet, drop the fraction in the quotient and add 1 to the whole number of strips. The extra width in such cases is either cut off or turned under.

In carpeting and papering, it is usually necessary to allow for a certain waste in matching the figures of patterns; and often carpets may be laid with less waste one way of the room than the other. Dealers charge for all goods furnished, regardless of waste. The price per yard for making refers to the total number of yards sold. Final quantities over a yard and under ¼ call ¼; over ¼ and under ½ call ½, etc.

291. To find the number of tons of coal when the quantity is expressed in pounds.

EXAMPLE.—How many tons of coal in 18424 lbs.?

OPERATION.

18424 lbs. ÷ 2000 = 9.212.

EXPLANATION.—Since there are 2000 lbs. in 1 ton there will be as many tons in 18424 lbs. as 2000 are contained times in 18424, or 9.212.

Rules.—2. *Divide the total number of pounds by 2000 the quotient will be in tons and decimal of a ton. Or,*

2. *Cut off three places from the right and divide by 2, the quotient will be in tons and decimal of a ton.*

292. To find the number of tons, cwts. and lbs. in a quantity of coal expressed in pounds.

EXAMPLE.—How many tons, cwts. and odd pounds are there in 20850 lbs. coal?

OPERATION.

2) 20	8	50
10 T.	8 cwt.	50 lbs.

EXPLANATION.—The first two figures, 50, represent the remainder below hundreds; the third figure, 8 cwt., and in the 20 thousand there will be as many tons as 2 thousand are contained times in 20 thousand, 10 times or 10 tons. Hence 20850 lbs. = 10 T., 8 cwt., 50 lbs.

Rule.—*Cut off two places for pounds, three places for cwts. and pounds, and divide thousands by 2 for tons.*

Coal miners and wholesale dealers usually render manifests and inventories in tons, cwts. and lbs.; retailers usually sell by the ton and fraction of a ton.

293. To find the number of board feet in a board, plank or timber.

EXAMPLE.—Find the total number of board feet in 1 board 12 in. wide, 1 in. thick and 17 feet long; 1 scantling 3 in. by 4 in., 15 feet long; and 1 joist 4 in. by 6 in., 20 feet long.

OPERATION.

$$\frac{12 \times 1 \times 17}{12} = \frac{204}{12} = 17 \text{ feet.}$$

$$\frac{3 \times 4 \times 15}{12} = \frac{180}{12} = 15 \text{ feet.}$$

$$\frac{4 \times 6 \times 20}{12} = \frac{480}{12} = 40 \text{ feet.}$$

Total, 72 feet.

EXPLANATION.—1 board foot is 12 inches wide, 12 inches long and 1 inch thick, hence for every 12 inches running length *1 inch thick* there is 1 board foot; and therefore in 17 running feet there are 17 × 1 board foot or 17 board feet. For each running foot 1 in. thick of the scantling there is ⅓ of a foot board measure (4 inches ÷ 12 inches = ⅓), and for the 3 inches of thickness, 3 times ⅓ foot, or 1 foot, and for 15 feet running measure, $15 \times \frac{3 \times 4}{12} = 15$ feet. For each running foot, 1 in. thick, of the joist there is ½ of a foot board measure (6 inches ÷ 12 inches = ½ foot), and for the 4 inches of thickness, 4 times ½ foot or 2 feet; and for 20 feet running measure, $20 \times \frac{4 \times 6}{12} = 40$ feet. Total, 17 feet + 15 feet + 40 feet = 72 feet.

Rule.—I. *To find the number of board feet in each running foot, divide the product of the thickness and width in inches by 12, and the quotient will be the board feet in 1 running foot.*

II. *To find the number of board feet in 1 piece, multiply the number of board feet in 1 running foot by the length in feet.*

Business Rules.—1. *The product of the number of pieces, width and thickness in inches and length in feet divided by 12 will give the number of board feet. Or,*

2. *Indicate the product of the pieces, width, thickness and length, and drop or cancel 12 or the factors of 12. The product of the remaining factors will be the board feet required.*

294. Finding difference in time.

Two methods are employed in finding the difference in time between any two given dates. One is to find the exact number of days, based on 365 days to the year, and the other by compound subtraction, based on 360 days to the year.

295. To find the exact number of days.

EXAMPLE.—Find the exact number of days from March 11th, 1889, to May 28th, 1890.

OPERATION.

Mar. 11th, 1889, to Mar. 11, 1890 =		365 ds.
Days remaining in	March,	20 ds.
Days in	April,	30 ds.
Days used in	May,	28 ds.
	Total,	443 ds.

EXPLANATION.—Since legally the day of date is excluded and the day of maturity is included for one year, calculating from March 11th, 1889, to March 11th, 1890, the exact number of days is 365. To these add the remaining days in March, which are 20, the days in April, 30, and the days in May up to and including the day of maturity, 28, and obtain a total of 443 days.

Rule.—*Exclude the day of date and count the remaining days in that month. Add to this the exact number of days through each month which the time extends and the number of days in the month of maturity up to and including the day of maturity.*

Should the day of date and the day of maturity be in the same month, subtract the first date from the last and the difference will be the time required.

296. To find the difference in time by compound subtraction.

EXAMPLE.—How many years, months and days from October 14, 1885, to June 5, 1891.

OPERATION.

yrs.	mos.	ds.
1891	6	5
1885	10	14
5	7	21

EXPLANATION.—Write the dates so that years, months and days stand under each other. Since 14 days cannot be subtracted from 5 days, take 1 month from the 6 months, leaving 5 months; 1 month = 30 days and 5 days added = 35 days; 35 days — 14 days = 21 days, which write under the column of days. Next, for the same reason, change 1 year to months. 1 year = 12 months, and adding the 5 months = 17 months. 17 months — 10 months = 7 months. 1890 years — 1885 years = 5 years; therefore, the time = 5 years, 7 months, 21 days.

Rule.—*Arrange the dates so that years, months and days stand under each other; then subtract as in subtraction of denominate numbers, allowing 30 days for a month and 12 months for a year.*

MISCELLANEOUS EXAMPLES.

297. *1.* How many perches of masonry in a wall 170 ft. long, 32 ft. high and 1½ ft. thick?

2. A bin is 18 ft. long, 4½ ft. wide and 8 ft. high. How many cubic feet in the bin?

3. At 15¢ per cu. yd. how much will it cost to excavate a cellar 85 ft. long, 43 ft. wide and 14 ft. deep?

4. What must be the length of a vat which is 4 ft. wide and 6 ft. high to contain 100 hhds. of water of 63 gals. each?

5. The hay filling a mow, 22 ft. long by 18 ft. wide and 4 ft. deep is sold at $17.50 per ton, and $1 per ton extra for drawing. Find the total cost if 10¢ per load or part of load is charged the buyer for weighing, each load averaging 1½ tons. (Rule 3, page 96.)

6. How much lumber in 18 pieces 5 in. × 7 in., 19 ft. long?
$\overset{1\frac{1}{2}}{\cancel{18}} \times 5 \times 7 \times 19 = 997\frac{1}{2}$, or 997½ ft. Observe that 12 cancels 18 one and one-half times, and $1\frac{1}{2} \times 5 \times 7 \times 19 = 997\frac{1}{2}$.

7. Find the exact number of days from May 1 to June 1.

8. How many gallons will a tank hold 4 ft. long, 3½ ft. wide and 1 ft. 10 in. deep?

9. How many board feet in 4 joists 3 in. × 5 in., 17 ft. long?

10. How many tons of hay on a wagon load 9 ft. long, 8 ft. wide and 6 ft. high?

11. How many perches of stone will be required to inclose a field 32 rods long, 24 rods wide with a wall 4½ ft. high and 2½ ft. thick?

12. Find the time by compound subtraction from Jan. 15th, 1890 to Dec. 14th, 1890. What is the exact number of days.

13. How many common bricks, laid in mortar ¼ in. thick, will be required to build 15000 cu. ft. of brick work ?

14. How many yards of carpeting 1 yd. wide will be required to carpet a room 24 ft. by 42 ft., to be laid with the strips running the longer way of the room, and no waste?

15. How many tons, cwt. and lbs. short ton weight in 10¾ long tons of coal?

16. How many barrels of water will a tank hold that contains 87318 cubic inches?

17. John C. Dow loaned Philip Smith a sum of money on interest from June 19th, 1888 to Dec. 20th, 1890. Find the exact number of days for which interest was charged.

18. How many cubic feet in a stone wall 195 ft. long, 4½ ft. high and 1½ ft. thick?

19. How much coal in tons, cwt. and pounds in 4 car loads weighing, respectively, 20140 lbs., 22820 lbs., 21400 lbs., 23850 lbs.? How many tons and decimal of a ton?

20. How many Milwaukee bricks will be required to build 4500 cu. ft. masonry laid in courses of mortar ¼ in. thick?

21. How many yards of Ingrain carpeting 1 yd. wide, laid lengthwise of the room, will be required to cover the floor of a room 22 feet long by 17 ft. 4 in. wide, the waste being 6 in. on each strip? How many yards of Brussels ¾ yd. wide will cover the same room, waste 1 foot per strip?

22. How many days from Feb. 14th, 1885 to June 16th, 1890? What is the time by compound subtraction?

23. How many common bricks, laid in mortar ¼ in. thick, will be required to erect a building 120 ft. long, 85 ft. wide and 22 ft. high, outside measurement, the walls to be 18 in. in thickness, and no allowance for openings?

24. Making an allowance of 2 feet waste for each strip, how much will it cost to carpet a room 32 feet long and 17½ feet wide, the strips to run lengthwise, with Brussels carpeting ¾ yard wide, at $1.35 per yard and 5¢ per yd. for making?

See note, top of page 97.

25. How many cubic yards of sand in a pile of sand 9 yards long, 6 yards wide and 4 yards high?

26. Find the number of Baltimore bricks required to erect a retaining wall 75 ft. long, 8 ft. high and 18 in. thick, laid in mortar ¼ in. thick.

27. How many board feet in a timber 2 in. × 6 in., 18 ft. long?

28. A hall 7½ ft. wide and 21 feet long is covered with linoleum at $1.10 per square yard. If 5¢ per square yard be charged for laying it, how much did it cost to cover the hall?

29. How many cubic feet in a pile 190 in. long, 20 in. wide and 18½ in. high?

30. A note dated Jan. 1st, 1888, is paid on Oct. 17th, 1889, with interest for the exact number of days. Find the number of days for which interest was computed.

31. How much will it cost to carpet the following rooms with ¾ yd. carpeting at $1 per yard, including the making, which is 3¢ per yard: 1 room 22½ ft. long and 14 ft. wide, and 1 room 17¾ ft. long and 9 ft. wide, the strips to run the long way of each room, with a waste of 8 in. on each strip.

32. What will be the cost of building the walls of a block, 140 ft. long, 66 ft. wide and 47 ft. high, outside measurement, at $1.45 per perch of 16½ cu. ft., if the walls are 18 in. thick and no allowance be made for openings?

33. What will be the cost of the plank in a 2 in. plank walk laid in front of a lot 450 feet on the street, the walk to be 5½ ft. wide if the plank costs $18 per M feet?

34. A room 21 ft. 8 in. long, 14 ft. 10 in. wide is to be carpeted with ¾ yd. wide carpeting at $1.40 per yard. If the strips are to run the width of the room, how many yards will be required, the waste being 8 in. on each strip, and what will be the total cost, including 5¢ per yard for making?

35. At $2.25 per perch, how much will it cost to build a retaining wall for a building, the length of which is 60 ft., and width 18 ft., the wall to be 14 feet high from foundation and 20 in. thick?

36. Find the amount of the following bill, prices by the M feet:

6 pcs. 2 in. × 6 in.—19 ft. = ———, @ $12.
10 pcs. 3 in. × 4 in.—20 ft. = ———, @ $15.
100 pcs. 8 in. × 10 in.—14 ft. = ———, @ $20.

In charging or billing lumber the *pieces* are entered first, then the *thickness* and *width*, then the *length*. For example, in recording 5 pieces 4 in. thick by 6 in. wide and 20 ft. long, the form would be thus: 5 pcs. 4 in. × 6 in.—20 ft., and would be called off by a salesman, "*5 Four-by-Sixes—20 feet.*" "Four-by-Sixes" being the name by which he selects and sells stock; such as "Three-by-Fours," "Eight-by-Tens," etc.

Instead of writing *inches* and *feet*, lumber dealers use the indices, (″) for inches and (′) for feet; thus 3 in. by 4 in.—17 ft. long is written 3″ × 4″—17′.

On account of uneven ends in lumber all fractional parts of a foot under ½ are dropped, if ½ or more and less than 1 ft., it is called ½, except in cherry, black walnut, etc., where the price is 15¢ per foot and upwards.

37. What will 10 pcs. 3″ × 4″—20′ cost at $15 per M feet?

38. Find the total number of board feet in following charge:

5 pcs. 3″ × 4″—16′;
8 pcs. 2″ × 6″—17′;
9 pcs. 4″ × 6″—20′;
10 pcs. 5″ × 7″—12′;
15 pcs. 8″ × 10″—20′.

Cancel 3, one of the factors of 12 out of 15, in the last item, and 4, the other factor of 12 out of 20, and the product of 5 × 8 × 10 × 5 = 2000 ft., the number of board feet in the 15 pieces, *eight-by-tens*—20 feet long.

39. Correct by scanning the extensions in following charge:

8 pcs. 3″ × 4″—20′ = 160′;
10 pcs. 2½″ × 8″—18′ = 300′;
20 pcs. 4″ × 6″—17′ = 340′;
12 pcs. 8″ × 10″—14′ = 1120′;
17 pcs. 6″ × 8″—19′ = 1292′.
Total, 3212′.

40. Find the amount of the above charges, at $15. per M feet.

PERCENTAGE.

298. Percentage is a term applied to computing by hundredths.

The Essential Elements of Percentage are the Base, the Rate and the Percentage.

299. Per Cent. is an abbreviation of the Latin phrase *per centum,* which signifies by the hundred.

300. The **Sign of Per Cent.** is %. It is used to denote per cent. and is read *per cent.*

301. The **Base** is the number upon which the per centage is computed, or, of which a certain number of hundredths are taken.

302. The **Rate** denotes how many hundredths of the base are to be taken, and is usually expressed as a decimal.

303. The **Percentage** is the result obtained by taking a certain per cent. of the base, or the product obtained by multiplying the base by the rate.

304. The **Amount Per Cent.** is 100 per cent. increased by the rate, or 1 *plus* the rate expressed as a *decimal.*

305. The **Difference Per Cent.** is 100 % decreased by the rate, or 1 *minus* the rate expressed as a decimal.

306. The **Amount** is the *sum* of the base and the percentage, or the *product* of the *base* multiplied by the *amount per cent.*

307. The **Difference** is the *remainder* after deducting the percentage from the base, or the *product* of the *base* multiplied by the *difference per cent.*

308. The Rate is correctly expressed as a decimal when used to indicate computations; thus, 12½ % is written .12½ and read *twelve and one-half hundredths;* or as .125 and read *one hundred twenty-five thousandths.*

309. Per cent. may be expressed as a common fraction by writing the rate per cent. for the *numerator* and 100 for the denominator; this reduced to its lowest terms would equal the fractional part of the base. Thus $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$ of the base, or $\frac{1}{8}$ of 100 %.

310. A Common Fraction may be changed to an equivalent *per cent.* by dividing the numerator by the denominator, pointing off at least two decimal places in the quotient.

Every rate per cent., being as many hundredths, requires at least two decimal places when expressed as a decimal; hence, if the rate per cent. be less than 10, prefix a cipher to the rate; as, 2 % = .02.

311. To read per cent.

Call the first two figures *per cent.* when followed by the character %, and add the name of the fraction; thus 18¾ % is read 18¾ per cent.

When expressed decimally as .1875 or .18¾, read the first two figures as hundredths and the added places as fractions.

312. Since *per cent.* is any number of *hundredths,* it may be expressed either as a *decimal* or as a *common fraction,* and the table of aliquot parts can be used with little variation and to great advantage in many operations in *percentage.* Hence, the rules given under Special Applications may be applied in this subject.

COMPARATIVE PER CENT. TABLE.

Sign.	Decimal.	Fraction.	Part of Base.	Sign.	Decimal.	Fraction.	Part of Base.
1 %	= .01	= $\frac{1}{100}$	= $\frac{1}{100}$	6¼ %	= .0625	= $\frac{6\frac{1}{4}}{100}$	= $\frac{1}{16}$
4 %	= .04	= $\frac{4}{100}$	= $\frac{1}{25}$	8⅓ %	= .08⅓	= $\frac{8\frac{1}{3}}{100}$	= $\frac{1}{12}$
5 %	= .05	= $\frac{5}{100}$	= $\frac{1}{20}$	12½ %	= .125	= $\frac{12\frac{1}{2}}{100}$	= $\frac{1}{8}$
10 %	= .10	= $\frac{10}{100}$	= $\frac{1}{10}$	16⅔ %	= .16⅔	= $\frac{16\frac{2}{3}}{100}$	= $\frac{1}{6}$
20 %	= .20	= $\frac{20}{100}$	= $\frac{1}{5}$	33⅓ %	= .33⅓	= $\frac{33\frac{1}{3}}{100}$	= $\frac{1}{3}$
25 %	= .25	= $\frac{25}{100}$	= $\frac{1}{4}$	66⅔ %	= .66⅔	= $\frac{66\frac{2}{3}}{100}$	= $\frac{2}{3}$
50 %	= .50	= $\frac{50}{100}$	= $\frac{1}{2}$	87½ %	= .87½	= $\frac{87\frac{1}{2}}{100}$	= $\frac{7}{8}$
75 %	= .75	= $\frac{75}{100}$	= $\frac{3}{4}$	125 %	= 1.25	= $\frac{125}{100}$	= 1¼
100 %	= 1.00	= $\frac{100}{100}$	= 1	150 %	= 1.50	= $\frac{150}{100}$	= 1½

313. The **Unit of Percentage** is 100 %, or the whole of the base.

314. The **Unit of Measure,** or fractional unit, is 1 %, or one of the one hundred equal parts ($\frac{1}{100}$) into which the base is divided.

315. To find the percentage, the base and rate being given.

EXAMPLE.—Find 38 % of 400 bushels.

OPERATION.

Base = 100 % = 400 bu.
Rate = 38 % = .38 of base.
Percentage = .38 × 400 bu. = 152 bu.

PROOF.

152 bu. = $\frac{152}{400}$ of 400 bu.
$\frac{152}{400} = \frac{38}{100} = .38 = 38$ %.

EXPLANATION. — The elements *given* are the base, 400 bushels, and the rate, 38%. The element *wanted* is the percentage in bushels. The percentage is the product of the base multiplied by the rate (Art. 303). Therefore 38 % of 400 bushels = .38 × 400 bushels = 152 bushels.

Proof.—As 1 bushel = $\frac{1}{400}$ part of 400 bushels, therefore, 152 bushels = $\frac{152}{400}$ of 400 bushels or $\frac{38}{100}$; and as $\frac{1}{100}$ = 1 %, $\frac{38}{100}$ = 38%.

Rules.—1. *Multiply the base by the rate expressed decimally. Or,*

2. *Take such a part of the base as the rate per cent. is a part of 100 %.*

When the rate per cent. is an aliquot part of 100%, find the percentage by rule 2nd. Thus :—For 50 % of 800 men take ½ of 800 men or 400 men. 33⅓ % of $450 = ⅓ of $450 = $150.

EXAMPLES BY ALIQUOTS.

316. What is

1. 5 % of 100 years ?
2. 12½ % of $3600 ?
3. 16⅔ % of 24000 lbs?
4. 33⅓ % of 450 bu. corn ?
5. 87½ % of 32000 cents ?
6. 50 % of 96 M ft. lumber ?
7. 66⅔ % of 1854 pwts. Troy ?
8. 6¼ % of 64 days ?
9. 8⅓ % of 4680 bricks ?
10. 125 % of $900.40.

EXAMPLES.

317. What is

1. 5 % of 80 lbs. ?
2. 20 % of $75.50 ?
3. 16⅔ % of 1500 men ?
4. 40 % of 500 ozs. ?
5. 30 % of 400 cents ?
6. 18 % of 300 days ?
7. 75 % of 785 cords ?
8. 22 % of 550 feet ?
9. 18 % of 200 miles ?
10. 25 % of $540.36 ?

11. A grocer bought 400 brls. apples for $1400. He sold 30 % of them for cash, 45% of them on time, and the remainder he sold at $5.00 per barrel. Find the value of the last sale.

12. In a city of 12500 population 12% of them were Germans, 20% English; 3% French, ⅘% Italian, and the remainder native-born Americans. How many of each nationalitv in the city?

13. A flock of sheep numbered 800 head. 50 % of them were sold for "prime" stock, 50 % of the remainder were sold as "seconds," 25% of the remainder died, and the remainder were turned out to pasture. How many were turned out to pasture?

14. In a cargo of mine run coal weighing 1250 tons, 30% of it sorted as grate coal, 15% as egg coal, 20% as nut coal, 12% as pea coal and the remainder as No. 4 coal. How many tons of each in the cargo?

15. A certain city official retiring from public life invests his fortune of $96500 as follows: 20% of it in Government bonds, 30% of it in a joint stock company, 50% of the remainder in city bonds, and the remainder in acre property. How many acres of land did he buy if he paid $40 per acre?

318. To find the base, the percentage and rate being given.

Example.—A selling agent's income for the month of January at 5 % commission was $840. What were his sales for the month?

OPERATION.

Base = 100% = sale,
Rate = 5% = .05 of sale,
1 % = unit of measure.
Percentage = $840.
As 5 % = $840,
1 % = $840 ÷ 5 = $168.
100 % = 100 × $168 = $16800.

PROOF.

5 % of $16800 =
.05 × $16800 = $840.

Explanation. — The elements *given* are the rate, 5 %, and the percentage, $840. The element *wanted* is the base. The unit of measure is 1 %. Since the rate and per centage are given it is evident that 5 % = $840, and that 1 % = ⅕ of $840 or $168. As 1 % = $168, 100 % = 100 × $168 = $16800.

Proof: 5 % of the base = .05 × $16800 = $840.

Rule—I. *Divide the percentage by the rate expressed as an abstract number, and the quotient will be 1 % of the base.*

II. *Multiply the quotient thus found by 100 and the product will represent the base.*

EXAMPLES.

319. *1.* 85 = 5% of what number?
2. 175 = 25% of what number?
3. $500 = 33⅓ % of how many dollars?
4. 25 lbs. = 6¼ % of how many lbs.?
5. 150 ozs. = 10 % of how many ozs.?

6. $9 = ¾ % of what amount ?
7. 1½ = 33⅓ % of what number ?
8. 2/7 = 50 % of what number ?
9. 25 % = 33⅓ % of how many per cent. ?
10. 33⅓ % = 50 % of how many per cent. ?

11. A house rents for $360 per year, which amount represents 8 % of its value. How much is the house worth ?

12. A grocer sold 55 bu. of potatoes in one month which amount represented 5½% of his stock of potatoes. How many bushels of potatoes had he at first?

13. From a cask of oil there leaked out 5.04 gallons, which was 8% of what it originally contained. How many gallons were there in the first place, and how many gallons remained in the cask?

14. A farmer sold 17 bu., 2 pks. and 1 qt. of tomatoes, which were 3% of his whole stock of tomatoes before selling. How many bushels of tomatoes had he at first?

15. From his bank account a man checked out $75.50, which amount was 30% of his account? How much had he in the bank at first?

320. To find the rate, the percentage and base being given.

Example.—A commission merchant sold goods amounting to $480, and received for selling the same $120. What was the rate of his commission ?

OPERATION.

Base = 100 % = $480
Percentage = $120 = commission.
Unit of measure = 1 % of $480 = $4.80.
$120 ÷ $4.80 = 25 times.
25 times 1 % = 25 %.
Or, 120/480 = the decimal, 120 ÷ 480 = .25.

Explanation. — The elements *given* are the base, $480, and the percentage, $120. The element *wanted* is the rate. The unit of measure is 1 %. Since $120, the percentage, is the product of the *base*, $480, multiplied by the *rate*, it is evident that this $120 is a certain *number of times* 1 % of $480. Now 1 % of $480 = $4.80, and therefore $120 will equal as many per cent., as $4.80 is contained *times* in $120, which are 25 times, and 25 × 1 % = 25%.

Rules.—1. *Divide the percentage by one per cent. of the base. The quotient will be the number of times 1 %. Or,*

2. *Express in fractional form and change to a decimal by dividing the numerator (percentage) by the denominator (base). The quotient will be in hundredths.*

EXAMPLES.

321. What per cent. of

1. 85 is 17?
2. 200 lbs. are 20 lbs.?
3. $600 are $12?
4. 500 men are 25 men?
5. 84 years are 7 years?
6. 250 ozs. are 12½ ozs.?
7. $62.50 are $1.50?
8. 75 bu. are 7½ bu.?
9. 195 qts. are 19½ qts.?
10. $1600 are $4.00?

11. A wool grower had 800 sheep at the beginning of the year and increased the flock 120 during the year. Find the per cent. of increase.

12. From a tank containing 120 gals. of water, 24 gals. evaporated in one week. Find the per cent. of evaporation.

13. In every 24 parts of gold jewelry sold by a pawn broker 6 parts were base metal. What per cent. of the 24 parts was gold?

14. A pile of 200 bushels of potatoes was buried in October, and when the pile was opened in May, 18 bushels were unsound. What per cent. of the whole was unsound?

15. If a man pays $12.50 for the use of $500 for one year what rate per cent. does he pay?

322. To find the amount per cent. and the amount, the base and rate being given.

EXAMPLE.—A jobber bought an invoice of goods at a cost of $1400. For how much must he sell them to make a profit of 15%?

OPERATION.

$$\begin{array}{ll} \text{Base,} & 100\,\% = \text{cost.} \\ \text{Rate,} & \underline{\ \ 15\,\%} = \text{gain.} \\ \text{Amt. per cent.,} & 115\,\% = \text{sale.} \\ \multicolumn{2}{l}{\text{Amount} = 1.15 \times \$1400 = \$1610.} \end{array}$$

PROOF.

$$\begin{array}{rcr} 100\,\% & = & \$1400 \\ \underline{15\,\%} & = & \underline{210} \\ 115\,\% & = & \$1610 \end{array}$$

EXPLANATION.—The essential elements *given* are the base and rate. The elements *wanted* are the amount per cent. and the amount. Since the base is 100% and the rate of gain is 15%, it is evident the sale must be 100% + 15% or 115% of the cost, and 1.15 × $1400 = $1610. The proof is based upon principles that the amount per cent. is the sum of 100% plus the rate, and the amount is the sum of the base plus the percentage, or the product of the base multiplied by the amount per cent.

Rule.—*Find the amount per cent. by adding the rate to 100%. Find the amount by multiplying the base by the amount per cent. expressed decimally.*

EXAMPLES.

323. *1.* The base is 252 and the rate 10 %, find the amount.

2. The base is 2150 and the rate 20 %, find the amount.

3. The base is 630 and the rate $33\frac{1}{3}$ %, find the amount.

4. The base is 546 and the rate $16\frac{2}{3}$ %, find the amount.

5. The base is 200 and the rate 25 %, find the amount.

6. A flock of sheep numbering 450, increased 10 % in one year. What was the number of the flock at the end of the year?

7. A city having a population of 12000 increased 60 % in ten years. What was its population at the end of the decade?

8. A school district in 1885 had school children numbering 450 entitled to public money. How many were there five years later if the rate of increase was 20 %?

9. In an orchard of 700 apple trees, 20 % additional trees were planted the second year. How many trees were there in the orchard at the end of the second year?

10. Goods which cost $1900 were sold at an advance of 18 %. What was the amount of the sale?

324. To find the difference per cent. and the difference, the base and rate being given.

EXAMPLE.—A farmer stored 800 bushels of potatoes in a cellar and lost 15 % of them by frost. How many bushels had he remaining?

OPERATION.

Base = 100 %
Rate = 15 %
Difference per cent. = 85 %
Difference = .85 × 800 bu. = 680 bu.

PROOF.

100 % = 800 bu.
15 % = 120 bu.
85 % = 680 bu.

EXPLANATIONS.—The essential elements given are the base, 800 bushels, and the rate, 15 %. The element wanted is the difference. Since the base = 100 % and the rate is 15 %, the difference = 100 % — 15 % = 85 %. Therefore the number of bushels remaining must be .85 of 800 bushels or 680 bushels.

Rule.—*Find the difference per cent. by subtracting the rate from 100%. Find the difference by multiplying the base by the difference per cent. expressed decimally.*

EXAMPLES.

325. *1.* If the base is 800 and the rate is 15 %, what is the difference?

2. If from a brood of 18 chickens 33⅓% be stolen, how many will be left?

3. What number will be left if 625 be diminished 20 %.

4. The deposits in a certain bank for the month of June were $10,000.00 and for July, Aug., Sept., Oct. and Nov., the deposits each month were 10% less than on the preceding month. What were the total deposits from July to Nov. inclusive?

5. From a cellar containing 25000 bushels of potatoes, 20 % were sold for cash, 40 % were sold on time, 5 % were lost by frost, and the remainder were carried over for the spring market. How many bushels were kept until spring?

326. To find the base, the amount or difference and the rate being given.

Example.—A man sold a horse for $230.00 and gained 15 % of the cost. What did the horse cost him?

OPERATION.

Cost = 100 %
Gain = 15 %
Sale = 115 %
115 % = $230
1 % = $ 2
100 % = $200

PROOF.

100 % = cost = $200
15 % of cost = 30
115 % of cost = $230

Explanation.—Since 100 % = cost and 15 % = gain, therefore 100 % + 15 % or 115 % of cost = sale. As 115 % = sale or $230.00, therefore 1 % = $230 ÷ 115 = $2.00; and 100%, or cost, = 100 × $2 = $200. Proof: As 15 % of the cost = gain— .15 × $200 = $30, and $200 + $30 = $230 = sale.

Rule.—*Divide the amount by the amount per cent., or the difference by the difference per cent., and the quotient will be 1 % of the base or cost. Multiply the quotient thus found by 100 and the product will represent 100 %, or the cost.*

EXAMPLES.

327. *1.* If the amount is 224 and the rate is 12 %, what is the base?

2. What number increased by 33⅓ % of itself will amount to $880?

3. A man's profit for one year was 12½%, when he had $2250. What was his capital at the beginning of the year?

4. An assignee paid off creditors to the amount of $750, which was 33⅓% less than the total indebtedness. What was the total indebtedness?

5. A stock of goods damaged by fire sold at 30% less than cost, or for $490.63. What was the cost?

MISCELLANEOUS EXAMPLES.

328. *1.* An article was marked in advance of the cost so as to net a profit of 25%. What per cent. should the selling price be discounted so as to net the cost?

2. What per cent. of 150% is 50% of the base?

3. An agent is allowed a discount of 40% from the list price for goods purchased from a manufacturer. What per cent. profit does the agent make if he sells the goods at list price?

4. An agent receives 33⅓% for sales and pays 25% of his commission for rent. What per cent. on sales does he net as profit?

5. 10% of 25% of 80% are how many per cent. of 100%?

6. An agent received a commission of 40% on sales. He spent 25% of his commission for rent and deposited the balance which was $60. What were his total sales?

7. An agent receives 25% for sales and 5% for collections. He remits $270 after deducting $300 commission on sales and his 5% on collections. How much did he collect and what were his sales?

8. Goods that are sold on time prices are offered at 10% less for cash. What per cent. on the cash basis does the purchaser pay for his credit?

9. 25% of 80% is 50% of how many per cent.

10. A man bought a house and lot and sold it at a profit of 20%. He invested this amount in acre property, and sold same at a profit of 33⅓%. With this amount he buys oil stock and sells at a loss of 10%. What was the amount of each investment, he having $8640 left?

11. If goods which cost $5000 were sold for $6900, what was the gain per cent.

12. If 38 gallons of pure vinegar and 2 gallons of water were put into a barrel holding 40 gallons, what per cent. of the whole was pure vinegar?

13. In an army of 45000 men 17000 men were killed, wounded and missing. What per cent. of the whole number was there remaining?

14. For the use of 75 bu. clover seed for one year a farmer paid 13½ bu. clover seed. What rate per cent. did he pay?

15. A city of 35000 population in 1880 had a population of 40775 in 1890. What was the per cent. of increase for the ten years?

16. What would be a dishonest dealer's gain in per cent. should he sell a pound of tea by Troy weight? What per cent. would he lose should he sell an ounce of tea by Troy weight?

17. A man lost 90 sheep during a storm, which he found to be 18% of his flock. How many were there in the whole flock at first?

18. A, B and C were partners having a joint capital of $4800. B has 20% more than A, and C has 33⅓% less than B. How much did each invest?

19. A gardener planted 4 bushels and 1 peck of potatoes and gathered from the field in the autumn 85 bushels. What was the per cent. of increase?

20. A fruit grower planted an orchard of 1500 fruit trees. There were 50% more cherry trees than peach trees, 50% less quince trees than peach trees, and as many apple trees as all of the other trees together. How many trees of each kind in the orchard?

PROFIT AND LOSS.

329. Profit and Loss treats of gains or losses in business transactions.

330. The **Profit** or gain is the sum left after deducting the net cost from a sale in excess of the cost.

331. The **Loss** is the sum of the sale deducted from the cost when the cost is in excess of the sale.

332. The **Gross** or **Full Cost** of an article is its first cost increased by all outlays.

333. The **Net Selling Price** is the gross price less all charges incident to the sale.

334. In ascertaining profit or loss, operations are usually performed by the rules of Percentage heretofore explained; but when the rate is a simple, common, fractional part of 100, it is more convenient to use the equivalent fraction than the decimal per cent.

335. Comparing the elements of Profit and Loss with those of Percentage, the *Cost* corresponds to the *Base;* the *Per Cent. of Gain* or *Loss* to the *Rate;* the whole *Gain* or *Loss* to the *Percentage;* the *Selling Price,* if at a gain, to the *Amount;* the *Selling Price,* if at a loss, to the *Difference.*

336. To find the profit or loss, the cost and rate being given.

EXAMPLE.—An agent bought a sewing machine for \$70.00 and sold it at a profit of 30%. What was his gain?

OPERATION.

Base = \$70.
Rate = 30%.
$.30 \times 70 = \$21.00$.

PROOF.

$\$21 = \frac{21}{70}$ of \$70.
$\frac{21}{70} = .30 = 30\%$.

EXPLANATION.—Since the base and rate are given to find the percentage (Art. 315) it is evident that the gain is such a part of the cost as the rate per cent. is a part of 100%.

Rules.—1. *Multiply the cost by the rate expressed decimally. Or,*

2. *Take such a part of the cost as the rate per cent. is a part of 100%.*

Any of the applications of Percentage may be worked by the United States money method, the same as the Interest method, page 43. The foregoing example may be worked thus:

OPERATION.

Cost = $70.00.
Gain on $1.00 = 30 cts.
70 × 30¢ = 2100¢, or $21.00.

EXPLANATION.—Since 30% represents 30¢ gain on each dollar, therefore, on $70 the gain is 70 × 30¢, or $21.00.

EXAMPLES.

337. Find

1. 5% gain of $350.
2. 6¼% gain of $800.
3. 33⅓% loss of $750.
4. 25% gain of $450.
5. 75% gain of $950.
6. 15% loss of $600.
7. ¼% gain of $16000.
8. 7½% gain of $12500.
9. 3⅓% loss of $1400.
10. 16⅔% gain of $1860.

11. A car load of coal costing $75 was sold at a profit of 15%. Find the amount of gain.

12. Since buying a cargo of sugar at a cost of $14500.00, the price depreciated 28%. How much was the loss?

13. Office furniture costing $185.00 was inventoried at the end of the year at 15% below cost. How much was the loss?

14. A manufacturer sold a lot of goods for $1260.00, thereby realizing a profit of 17½%. What was his gain?

15. Store fixtures amounting to $250.00 were inventoried the second year at 20% below cost, and the third year at 12½% below inventory. What was the loss the third year?

338. To find the cost, the gain or loss being given.

EXAMPLE.—By selling an article at 15% below cost, a loss was sustained of $45.00. What was the first cost?

OPERATION.

100% = cost.
15% = loss.
∴ 15% = $45.00,
1% = $45 ÷ 15 = $3.00.
100% = 100 × $3 = $300.

EXPLANATION.—Since 15% of cost = $45, 1% of cost = $45 ÷ 15 = $3. Since 1% of cost = $3, 100% or cost = 100 × $3 = $300. Proof: 15% of $300 = .15 × $300 = $45.

Rule.—*Divide the given loss or gain by the rate per cent. expressed as an abstract number, and the quotient will be 1% of the cost. Multiply the quotient thus found by 100 and the product will be 100%, or cost.*

EXAMPLES.

339. Find the cost if

1. \$15 = 7½% loss.
2. \$45 = 15% gain.
3. \$175 = 20% gain.
4. \$350 = 7% loss.
5. \$188 = 10% gain.
6. \$1.50 = 33⅓% loss.
7. \$150 = 75% gain.
8. \$37.50 = 3% loss.
9. \$450 = 20% gain.
10. \$850 = 17% loss.

11. A man sold a house at an advance over cost of 15%, and made \$304.56 profit. How much did the house cost him?

12. A merchant sold a quantity of butter at an advance of 16⅔% and realized a profit of \$360. He invested the proceeds in apples, which he sold at a loss of \$432. Find the cost of the butter and the selling price of the apples.

13. Bought 500 barrels of flour and sold it at an advance of 20%, thereby gaining \$600. Find the cost price per barrel.

14. James L. Norton bought a quantity of wool and sold it at a loss of 20%. He invested the amount received for the wool in hops which he sold at a gain of 20%, making a profit of \$128 on the last sale. Find the cost of each.

15. Barry & Green sold a quantity of potatoes at a loss of 10% and invested the proceeds in hay, which they sold at a gain of 10%, with a net loss of \$10 on both sales. Find the cost of each.

340. To find the rate of profit or loss, the cost and profit or loss being given.

EXAMPLE.—A reaper that cost \$350 was sold at a profit of \$105. Find the rate per cent. of gain.

OPERATION.

100% = cost = \$350.
gain = \$105.
1% of cost = \$3.50.
\$105. ÷ \$3.50 = 30 times.
30 × 1% = 30%.

EXPLANATION.—Since the gain or loss is a certain number of hundredths, the gain of \$105 will equal as many per cent. of the cost as 1% of the cost or \$3.50 is contained times in \$105, which equals 30 times, and 30 × 1% = 30%. Proof: 30% of cost = .30 × \$350 = \$105.

Rule.—*Divide the loss or the gain, as the case may be, by 1% of the base; the quotient will be times 1%, the product of which will represent the rate per cent. of gain or loss.*

EXAMPLES.

341. Find the rate per cent. of gain or loss when the cost is

1. $300 and the loss is $50.
2. $500 and the gain is $75.
3. $400 and the loss is $8.
4. $750 and the gain is $150.
5. $800 and the loss is $600.
6. $450 and the gain is $2.25.
7. $900 and the loss is $4.50.
8. $800 and the gain is $2.00.
9. $1750 and the gain is $750.
10. $1000 and the loss is $250.

11. An agent bought a machine for $42, and sold it at a gain of $28. What per cent. did he make on the cost? What per cent. on the sale?

Jobbers, retailers and agents usually calculate the rate per cent. of profit on sales. Manufacturers and investors calculate the rate on the cost.

12. An agent's sales for 1 month were $150 more than the cost price, which was $600. What per cent. profit did he make?

13. A farm of 160 acres, costing $50 per acre, was sold at a profit of $1600. What was the per cent. of gain?

14. Find the rate per cent. gained on the following: Molasses costing 60¢ at a gain of 15¢; sugar costing 4¢ at a gain of 1¢; coal costing $4.50 at a gain of 90¢; potatoes costing $155 at a gain of $11.62½.

15. A dry goods firm paid $800 for a job lot of goods which it sold at a loss of $160. With the proceeds they bought prime goods which they sold at a profit of $160. What per cent. was lost on the job lot and what per cent. was gained on the prime goods?

342. To find the cost, the selling price and the rate per cent. of profit or loss being given.

EXAMPLE.—A horse and carriage which sell for $240 net a gain of 20%. Find the cost.

OPERATION.

$$\begin{array}{rl} 100\% &= \text{cost.} \\ \underline{20\%} &= \text{gain.} \\ 120\% &= \text{sale} = \$240. \\ 1\% &= \$240 \div 120 = \$2. \\ 100\% &= 100 \times \$2 = \$200. \end{array}$$

EXPLANATION.—Since 100% = cost and 20% = gain, therefore, 120% = sale. Since 120% = $240, 1% of cost = $240 ÷ 120 = $2. And 100%, or cost, = 100 × $2 = $200. Proof: 20% of cost = .20 × $200 = $40, and $200 + $40 = $240.

Rule.—***Divide the amount of the sale by the amount per cent. and the quotient will represent 1% of cost. Multiply the quotient thus found by 100 and the product will represent 100%, or cost.***

EXAMPLES.

343. Find the cost price when the following sales produce accompanying gains:

1. $800 sale nets 33⅓%.
2. $750 sale nets 25%.
3. $400 sale nets 20%.
4. $950 sale nets 10%.
5. $2000 sale nets 100%.
6. Wood for $6.00 nets 20%.
7. Flour for $7.50 nets 4⅙%.
8. Cloth for 75¢ nets 25%.
9. Sugar for 6¢ nets 20%.
10. Calico for 9¢ nets 33⅓%.

MISCELLANEOUS EXAMPLES.

344. *1.* A grocer made the following sales: Potatoes, $852.00, at a profit of 20%; cheese, $255, at a profit of 6¼%; butter, $640.20, at a profit of 10%; and oranges, $225, at a loss of 10%. Find the net gain and the net per cent. of gain.

2. A man sold a piece of property at a gain of $85, which was 17% of the cost. How much did the property cost him?

3. By selling a cow for $75 the owner lost 33⅓%. How much did the cow cost him?

4. If ⅖ of an article sells for ⅗ of the whole cost of the article what is the gain per cent. on the amount sold?

5. A business man offered to sell his private residence for a certain price which, if received for the property, would have netted him a profit of 37½%. Not effecting a sale, he finally offered to discount the asking price 10%, which offer was accepted and the sale made. By the transaction he gained $950. How much did the property cost him?

6. By selling a quantity of wood for a certain sum, a dealer made 16⅔% on the cost. Had the wood cost him $240 more than it did, he would have lost 16⅔%. What was the cost of the wood?

7. A dealer in pianos marked an instrument 25% above cost, but sold it to a customer at a certain per cent. discount from the marked price and made a profit of 10% on the cost. What was the rate per cent. of discount?

8. Certain carpeting costs a dealer $1.20 per yard. What shall the selling price be so as to allow a discount of 10% to the purchaser and still make a profit of 25%?

9. A mine owner sells coal to a jobber at a profit of 20% on cost. The jobber sells the same cargo to a dealer at a profit of 16⅔% on the cost. The dealer retails it and on account of a reduced price sells it at $3.78 per ton and loses 10%. Find the cost per ton to the mine owner, the jobber and the dealer.

10. I sold my house to B and lost 10% of its cost; B expended $375 for repairs and sold it to C at 120% of its full cost to him; C expended $525 in enlarging the house, and then sold it for $6354, thereby making a profit of 20% of its full cost. How much did I pay for the property?

11. A quantity of stone sold for $7.00 and netted a profit of 16⅔%. Had it cost $2.40 more and sold at the same price, a loss of how many per cent. would have been sustained? How much did the stone cost?

12. A merchant bought a stock of goods costing him $800, one-half of which he sold for $650, and the remaining half he sold at a loss of 25% on cost. Did he make or lose, and how much? What was his per cent. of gain on first sale; what was his net gain or loss per cent.?

13. The profit of a merchant for the month of January was $300; for February 20% less than for January; for March 25% less than for February; for April 25% of the total profits from January to March inclusive. Find the profits for each month.

14. An invoice of white goods cost $5462.50. If freight on same was 1½% of purchase price, for how much must the whole be sold so as to net a profit of 16⅔% on the whole cost?

15. Graham & Co. bought a bill of raw cotton for $14000 and manufactured it into cotton goods at a cost of $8500. The office expenses and freight increased the total cost 15% of the cost of the raw material. If discounted at 10% for cash, for how much should the whole be sold on time so as to net a profit of 20%?

TRADE DISCOUNT.

345. Trade Discount is an allowance made by the seller to the buyer.

Discounts made as an inducement to buy are deducted from the list price when the sale is made.

Discounts offered as an inducement to pay before a debt is due are deducted when the bill is paid

346. The **List Price** is a fixed price for goods, and is usually based upon the retail price. It is customary in many branches of business for merchants and manufacturers to have *fixed price lists* of their goods, and when the market varies, instead of changing the price list, to change the *rate* of *discount.*

Business houses usually announce their terms upon their "bill-heads;" as, "Terms, 3 months, or 5% off for cash;" "Terms, 60 days, or 3% discount in 10 days," etc. When bills are paid before maturity, legal interest for the remainder of the time is usually deducted.

347. An **Additional Discount** is sometimes allowed which is based upon the extra amount that a customer may buy, and not upon the market as is the case with the first discount.

348. The essential elements are the same as in Percentage. The Base is the Gross Amount of the bill, except that with each additional discount the base is the difference per cent. or the difference, preceding each additional discount.

349. To find the net amount of a bill to render, the list price and the rate of trade discount being given.

EXAMPLE.—Find the net amount of bill to render for the following sale: Gould & Co., of Cleveland, O., sold Edw. R. Dunlap, Jan. 1, 1891, 5 bundles iron pipe, 500 ft., 28¢ per ft.; 25 valves, $2 each; 50 elbows, 20¢ each; 150 yds. Asbestus, 50¢ per yd. less 40% discount. Terms, 30 da., 2% 10 da.

BILL RENDERED.

CLEVELAND, O., Jan. 1, 1891.

Edw. R. Dunlap,

Bot. of Gould & Co.

Terms: 30 days, 2% 10 days.

5	bdls. 500 ft. Iron Pipe,	28¢	140		
	25 Valves,	$2.00	50		
	50 Elbows,	20¢	10		
	150 yds. Asbestus,	50¢	75		
			275		
	Disct. 40%.		110		
					165

EXPLANATION.—Since the 40% discount was an inducement to buy, therefore, the 40% of the $275 was deducted from the bill when rendered. On Jan. 31, Dunlap's bill of $165.00 will be due without any further discount. But if he pays the bill on or before Jan. 11th, he will be entitled to deduct 2% according to the terms stated, and he should then remit $165 less $3.30, or $161.70.

Rule.—*Deduct the discount on the sale from the gross amount of the bill, and the balance will be the net amount of the bill to render.*

350. To find the amount or balance to remit if a bill subject to a discount, be paid within a specified time.

Rule.—I. *Multiply the net amount of bill rendered by the net per cent. of discount; the product will be the amount of discount.*

II. *Deduct the discount from the amount of bill rendered, the balance will be the amount to remit.*

EXAMPLES.

351. *1.* Find the net amount of the following bill: 10 boxes glass, at $3.75 each, subject to a discount of 25%.

2. A wholesale dealer sells 150 organs at $225.00 each, less 45% discount. Find net amount of bill.

3. An invoice of silk footed in gross $1750, with 37½% discount to be deducted. What was the net amount rendered?

4. Martin W. Emmons bought of D. M. Jones, June 10th, 1891, on 30 ds., 2% 10 ds.: 2 pianos, at $450 each; 3 pianos, at $375 each, and 1 piano, at $250, less 40% discount on each from list prices; 5 organs, at $175 each, less 37½% discount from list price. Find net amount of bill to render; also find amount to be remitted if bill is discounted June 20th, at 2%.

5. Brown & Ellis sold H. D. King on 60 ds., 5% 10 ds., 10 family sewing machines, at $70 each, and 10 No. 8 sewing machines, at $80 each, less 45% from list prices. Find net amount of bill to render, and net amount to remit if paid within 10 days.

352. To find the net cost of an article, the list price and rate of trade discount being given.

EXAMPLE.—Find the net cost per foot of iron pipe, list price 28¢ per foot, discount 40%.

OPERATION.

100% = list.
40% = disct.
60% = net cost.
.60 × 28¢ = $16\frac{8}{10}$¢.

PROOF.

100% = list = 28¢.
40% = .40 × 28¢ = 11.2¢.
60% = cost = 16.8¢.

EXPLANATION.—The elements given are the base, 100%, and the rate, 40%. The element wanted is the difference per cent. of the base. Since 100% = list. and 40% = discount, the net cost equals 100% — 40%, or 60%; and 60% of the cost = .60 × 28¢ = $16\frac{8}{10}$¢.

Rule.—*Multiply the list price by the difference per cent. and the product will be the net cost.*

EXAMPLES.

353. *1.* Find the net cost of a box of glass listed at $3.75 and discounted 25%.

2. An organ listed at $225 was sold at a discount of 45%. What was the net cost of the organ to the purchaser?

3. A sewing machine listed at $65 is sold at a discount of 40%. Find the net cost to the purchaser.

4. What is the net cost of each of the following articles: pianos, listed at $450, $375 and $250, discount 40%; organs, listed at $175, discount 37½%?

5. Eisenbeirs & Co. bought goods listed as follows and discounted 30%: lead pipe, 60¢, 45¢, 25¢, 18¢; glass, 20¢, 15¢, 12¢, 8¢; steam valves, $4.00, $3.50, $2.50, $2.00 and $1.50. What was the net cost of each article, calling each fractional part of a cent 1¢ more in each result to cover breakage?

354. To find the net amount of a bill to render, when additional discounts are made.

EXAMPLE.—Find the net amount of bill to render for the following sale: Turney & Ames sold L. W. Green & Co., on 30 days, 2% 10 days. 75 cultivators, listed at $8.50, less 20%, 10% and 5%. Dated Corinth, Jan. 2, 1891.

BILL RENDERED.

CORINTH, Jan. 2, 1891.

L. W. Green & Co.,

Bot. of Turney & Ames.

Terms: 30 days; 2% 10 days.

	75 Cultivators,	$8.50	637	50		
		Disct. 20%	127	50		
			510			
		10%	51			
			459			
		5%	22	95		
					$436	05

EXPLANATION.—The gross amount of the bill, $637.50, is discounted 20%, or $127.50, leaving $510, which ordinarily would be the net amount to render. But as an *additional* inducement to buy, *this* amount is discounted 10%, leaving $459, which amount is also discounted 5%, leaving $436.05, the net amount of bill.

Rule.—I. *Multiply the amount to be discounted by the rate of discount, expressed decimally.*

II. *Deduct the first discount from the gross amount of the bill, and the remainder will be the amount to be discounted by the second rate of discount.*

III. *Deduct the second discount from the first remainder and the second remainder will be the amount to be discounted by the third rate of discount, if any.*

If there be no third rate of discount the second remainder will be the amount of the bill to render.

The amount of bill to render is extended into the outside or total column of the blank invoice.

Although the first discount and additional discounts are deducted in numerical order beginning with the greatest, the result will be the same in whichever order they are written.

EXAMPLES.

355. Find the net amount of bill to render for the following:

1. $875 less 30% and 20%.
2. $1725 less 30%, 10% and 5%.
3. $84.50 less 50% and 30%.
4. $500 less 20%, 10% and 5%.
5. $640 less 50%, 30% and 20%.
6. $200 less 25%, 10% and 5%.
7. $300 less 20%, 10% and 5%,
8. $300 less 5%, 10% and 20%.
9. $600 less 50%, 10% and 10%.
10. $4050.40, less 50% and 10%.

356. To find the net cost of an article, the list price, rate of discount and additional discounts being given.

Example.—Find the net cost of an article listed in a bill at $6.00, and the total bill discounted 40%, 25% and 10%.

OPERATION.

100% = list price.
40% = 1st discount.
60% = 2d base.
25% of 60% = 15% = 2d discount.
45% = 3rd base.
10% of 45% = 4½% = 3rd discount.
40½% = net per cent. of cost.
40½% of $6.00 = $2.43 net cost.

PROOF.

$6.00 = list price.
40% of $6.00 = 2.40 = 1st discount.
$3.60 = 2d base.
25% of $3.60 = .90 = 2d discount.
$2.70 = 3d base.
10% of $2.70 = .27 = 3d discount.
$2.43 = net cost.

Explanation.—Represent the list price by 100%. Successively discount each base, the first by 40%, the second by 25%, and the third by 10%. The remainder, 40½%, is the difference or net per cent. of cost; and 40½% of $6.00 equals $2.43. The proof of this is to deduct from $6.00, 40% of itself, which equals $6.00 — $2.40 = $3,60; 25% of $3.60 = 90¢, and $3.60 — .90 = $2.70; 10% of $2.70 = 27¢, and $2.70 — .27 = $2.43.

A short cut to find the net per cent. is to deduct each rate from 100, and multiply together the three remainders. Thus, for the above discounts of 40%, 25% and 10%, subtracting each from 100 gives: 100% — 40% = 60%; 100% — 25% = 75%; 100% — 10% = 90%, and .60 × .75 × .90 = .405, or 40½%.

Rules.—1. *Multiply the list price by the difference or net per cent. and the product will be the net cost. Or,*

2. *Subtract each rate per cent. from 100%. The product of the remainders will be the net per cent.; then multiply the list price by the net per cent. as in rule 1.*

EXAMPLES.

357. *1.* Find the net cost of an article listed at $12.00 and subject to a discount of 20%, 10% and 5%.

2. What is the net cost of 1 steam valve listed at $4.50 and discounted 30%, 20% and 10%.

3. A piano listed at $450 was sold at a discount of 40% and 10%. If the freight was $5.50 and the drayage, $3.50, what was the net cost of the piano?

4. Find the net cost of articles listed as follows: Lead pipe, 65¢, discount 40% and 10%; iron pipe, 30¢, discount 45% and 20%; bath tubs, $12, $10 and $8, discount 20% and 10%.

5. What was the list price of an article that, when discounted 40% and 10%, the net cost was $4.05?

MISCELLANEOUS EXAMPLES.

358. *1.* Find the net amount of a bill to render for the following sale: Theodore W. Cooper sold Jarvis & Co., 30 days, 2% 10 days, 20 pianos at $325 each, 10 organs at $125 each, less 40%. Find amount to be remitted if paid within 10 days.

2. The net amount of a bill rendered by G. W. Chapman & Co., of Chicago, to L. M. Bannister, Olean, was $812.60, under date of March 1st, 1891; terms, 30 days, 2% 10 days. How much was remitted if the bill was paid March 11th, 1891?

3. Find the gross amount of the following bill, which was made out at list prices, discounted 40% when rendered and the net amount *remitted* after deducting 2% for paying 10 days after date: Burch & Co. remitted Theodore W. Cooper $382.00 in payment invoice Jan. 5, less 2%.

4. Find the net cost of 1 steel engraving, listed at $18.00 and discounted 40% and 10%; 1 piece Rogers' statuary, listed at $40 and discounted 35%, 20% and 10%; 1 piece Japan ware, listed at $150 and discounted 60%, 25%, 20% and 5%.

5. The net price of a piano was $192. Find the list price if the rates of discount were 40% and 20%.

6. For the net amount of a bill rendered there was remitted $264.60 in payment, less 2%. What was the gross amount, if the rates of discount were 50% and 10% ?

7. Find the total amount to remit for the following bills, if discounted as follows: Morgan & Co., $350 less 2%; Graham & Elder, $950 less 3%; and T. C. Brown, $600 less 5%.

8. To the net cost of an article, $1.50 was added for freight, making the total net cost $6.00. What was the list price, the rates of discount being 50% and 5% ?

9. Which is cheaper, to buy goods at a discount of 60%, 20% and 10%, or at 50%, 40% and 10% ?

10. To the cost of 1 doz. steam valves, $3 were added, making the total net cost $30 per doz. Find the list price, the discount being 40%.

11. Goods cost net $3.12 per dozen. Find the list price and the second discount, if the first discount was 40% and the equivalent discount was equal to 52% off ?

12. If an article is discounted 50%, 25% and 10%, what would be an equivalent discount? Prove it by finding the net cost of an article, the list price of which is $8.00, by the serial discounts and by an equivalent discount.

13. One drummer offers to sell $500 worth of iron pipe at a discount of 40% and 10%, and another offers 33⅓% and 25%, discount. Which is the better rate, and what is the difference expressed in dollars?

14. If goods are listed at $12 per dozen and I buy at a discount of 40% and 33⅓%, what discount must I make from the list price in addition to 40% and still make 25% profit on my net cost?

15. F. D. Babcock & Co. sold Barnes & Co. on ℅, 30 days, 2% 10 days, 50 cultivators, less 33⅓% from the list price of $7.50 each, and bought of them on 30 days, 2% 10 days, 2500 ft. iron pipe at 26¢, less 45%. Ten days later a settlement was made, each party discounting his bill. Find the net amount rendered of each bill, the net amount settled for and the amount of error that Babcock & Co. made, who deducted the amount due them from the amount owed Barnes & Co. and remitted $100.35.

COMMISSION.

359. Commission is the sum charged by an agent or factor for transacting business for his principal.

The agent is the person who transacts the business for another. The principal is the person for whom the business is transacted.

When the agent buys or sells merchandise for others he is called a commission merchant.

360. A **Consignment** is a quantity of goods shipped by one party to another to be sold by the one receiving it for the shipper.

The shipper is called the consignor; the one to whom the goods are shipped is called the consignee.

361. The **Gross Proceeds** of a sale or collection is the total amount received by the agent before deducting his charges.

362. The **Net Proceeds** is the balance due the consignor after all charges are deducted.

363. An **Account Sales** is a detailed statement of the sale and charges rendered by the agent.

The agent's commission for selling is computed on the gross proceeds, and for purchases on the prime cost.

364. Computations in Commission involve the general principles of Percentage and may be compared as follows:

The Prime Cost or Gross Sale corresponds to the Base; the Rate Per Cent. of Commission corresponds to the Rate; the Commission corresponds to the Percentage; Net Proceeds corresponds to the Difference; Gross Proceeds, or Total Remittances to agent, corresponds to the Amount.

365. To find the commission, the cost or selling price and the rate per cent. of commission being given.

EXAMPLE.—An agent sells goods to the amount of $800 and charges 5% commission. What is the amount of his commission?

OPERATION.

Sale = $800.
Rate = 5%.
Comm. = .05 × $800 = $40.

PROOF.

$\$40 = \frac{40}{800}$ of $800 = .05, or 5%.

EXPLANATION. — The elements given are the base and rate. The element wanted is the percentage. Proceed as in Article 315, page 105.

General Rule.—*Multiply the cost price or the selling price by the rate per cent. of commission, the product will be the commission.*

Special Rule.—To calculate commission for 10%, cut off *one* place from the right of dollars; for 5%, take ½ of the commission at 10%; for 2½%, ¼ of the commission at 10%; for 1%, cut off two places from the right of dollars; for ½%, take ½ the commission at 1%; for ¼%, ⅛%, etc., take ¼, ⅛, etc., of the commission at 1%.

EXAMPLES.

366. What is the commission on

1. $8750.50 at 10% ?
2. $5114 at 5% ?
3. $750.80 at 10% ?
4. $1460 at 7½% ?
5. $600.98 at 5% ?
6. $2250 at 2½% ?
7. $16807 at 1% ?
8. $12608 at ½% ?
9. $4750 at ¼% ?
10. $68452.80 at ⅛% ?

11. An agent sells for his consignor, goods amounting to $850.63, at 5% commission. Find the amount of his commission.

12. L. M. Brandon sells for Jones, Kasson & Co. for cash: 5 reapers, listed at $750, with 30% and 20% off; 10 cultivators, listed at $15.00, with 40%, 10% and 5% off; 1 keg nails, $2.50 net. Find Brandon's commission at 5% on net sales.

13. An agent sells for a manufacturer in one month, goods to the amount of $2851.75, at a commission of 5%, and purchases for him raw material amounting to $5580, at a commission of 2½%. Find the total amount earned by the agent.

14. A real estate agent sells a piece of outlying city property for W. B. Gould to J. H. Ingraham for $30000. He charges Gould 5% for selling. If his livery bill and advertising cost him $10.50, what was his net profit on the transaction?

15. A selling agent receives orders to buy in the market a quantity of cotton and to sell the same in job lots. He buys $8500 worth at a commission of ½%, and sells the same for $9250.75, at a commission of 2½%. After deducting the total amount due him for buying and selling from the sale he invests the remainder of cash on hand in dry goods, and ships the same to his principal. How much is due the agent for commission on the dry goods, if he charges 1½ ?

367. To find the investment or gross sales, the commission and rate per cent. of commission being given.

EXAMPLE.—If a selling agent's commission is 5%, what amount of sales did he make in a month when his commission for that time amounted to $175.10?

OPERATION.

100% = sales.
5% = rate of commission.
$175.10 = commission.
5% = $175.10.
1% = $35.02.
100% = $3502.

EXPLANATION.—Since 5% represents 5 one-hundredths of the sales, therefore 5% = $175.10, and 1% = $175.10 ÷ 5 = $35.02. 100%, or total sales = 100 × $35.02 = $3502.

Rule.—*Divide the commission by the rate, the quotient will be 1% of the sales. Find 100% of sales by moving the decimal point in the quotient two places to the right.*

EXAMPLES.

368. Find the sales when

1. 5% = $85 commission.
2. 2½% = $900 commission.
3. ⅛% = $125 commission.
4. ¼% = $320 commission.
5. 2½% = $175 commission.
6. 5% = $48.10 commission.
7. 10% = $128.75 commission.
8. 5% = $486 commission.
9. ⅛% = $12.50 commission.
10. ½% = $37.50 commission.

369. To find the cost price of a purchase and the commission, when the amount remitted to the agent by the principal and the rate are given.

EXAMPLE.—A manufacturer remits a New York purchasing agent $2100 with which to buy a bill of cotton goods. How much shall the agent invest and what is the amount of his commission, if he charges 5% for making the purchase?

OPERATION.

100% = investment.
5% = commission.
105% = remittance.
105% = $2100.
1% = $20.
100% = $2000, cost of goods,
5% of cost = $100, commission.
Proof: 105% = $2100, remittances.

EXPLANATION.—Since for every dollar of cost the agent will charge 5¢ commission, therefore, for every $1.05 remitted the agent can invest $1.00. And he can invest $1.00 as many *times* as $1.05 is contained in $2100, or 2000 times. Let 100%, then, equal cost, 5% = commission, then 105% equals remittance. As 105% = $2100, 1% = $2100 ÷ 105 = $20; and 100% = 100 × $20 = $2000, or investment; and 5% = 5 × $20 = $100, or commission.

Rule.—I. *Divide the remittance by 100% plus the rate of commission, and multiply the quotient thus obtained by 100. The product will be the cost price of a purchase or the investment.*

II. *Subtract the cost price or the investment from the remittance, and the remainder will be the commission*

EXAMPLES.

370. Find amount to invest and the commission when the following remittances and rates are given.

1. $1751 at 3%.
2. $1050 at 5%.
3. $205 at 2½%.
4. $3225 at 7½%.
5. $4010 at ¼%.
6. $550 at 10%.
7. $1545 at 3%.
8. $703.50 at ½%.
9. $1230 at 2½%.
10. $1498 at 7%.

11. If an agent's commission is 5% on purchases, how much should he invest for his principal who remits $1551.90 to cover both investment and commission on same?

12. How many pounds of wool at 21¢ per pound can be bought for $1764 after allowing an agent 5% on purchase price?

13. A real estate agent receives $8302.50 to invest in western land. How much per acre did he pay if his commission on investment was 2½% and 150 acres were purchased?

14. A commission merchant remits to his St. Louis agent $1785.00 with which to invest in western wheat and to pay the agent's commission of 5% on purchase. How many bushels of wheat, at $1.25 per bushel, can tho agent buy?

15. An agent sold for his principal goods to the amount of $9000, upon which he charged a commission of 2½%. He invested the remainder, after deducting his commission of 2% for buying, in dry goods. Find the cost of the dry goods bought.

371. To find the net proceeds, when the sales, charges and rates per cent. for commission, insurance and guaranty are given.

EXAMPLE.—Find the net proceeds of the following sales by O. M. Duncan & Co. for E. L. Brewster & Co., Jan. 5, 1891: 10 brls. flour, @ $6.00; Jan. 10, 100 brls. flour, @ $5.75; Jan. 20, 90 brls. flour, @ $5.70. Charges, freight, $20.00; insurance, ¼%; guaranty, ¼%; commission, 2¼%. Dated Columbus, Jan. 30, 1891.

ACCOUNT SALES.

COLUMBUS, Jan. 30, 1891.

Account Sales
By O. M. Duncan & Co.
For acct. E. L. Brewster & Co.

1891. Jan.	5	10 brls. Flour,	$6.00	$ 60.		
	10	100 brls. Flour,	$5.75	575.		
	20.	90 brls. Flour,	$5.70	513.	1148	
		Charges:				
		Freight, $20.00. Insurance, ¼%, $2.87.				
		Guaranty, ¼%, $2.87. Comm. 2¼, $25.83.			51	57
		Net proceeds,			$1096	43

EXPLANATION.—The total sales amount to $1148, which represents 100%, or the base. ¼% of the base, or $2.87, represents the insurance, the same amount for the guaranty, and 2¼% the commission, or $25.83. The total charges is the sum of the insurance, guaranty, freight and commission, which are $20. + $2.87 + $2.87 + $25.83, or $51.57. The total sales, less charges, or $1148. — 51.57 = $1096.43, net proceeds.

Rule.—*Deduct the total charges from the total sales, the difference will be the net proceeds.*

Charges in the commission business are outlays incident to the purchase or sale. Guaranty, insurance and inspection are, like commission, calculated at a certain rate per cent. on the gross sales. Freight, cartage and storage are fixed prices by the article or quantity, according to distance and time.

EXAMPLES.

372. *1.* Find the net proceeds of the following account sales: June 1, 1891, 100 brls. apples, @ $6.00, less freight, $38; commission, 5%.

2. What are the net proceeds of 500 brls. pork @ $20, less charges: freight, 50¢ per barrel; insurance, ¼%; guaranty, ¼%; commission, 2½%?

3. Account sales by L. M. Goswold, Chicago, for account of Pemberton & Gunniss: 300 brls. pork, $18.00; 100 brls. beef, $17.50. Charges: freight, 50¢ per barrel; commission, 2½%. Find the net proceeds.

4. Account sales by Gordon, McMillan & Co., Cleveland, for account of D. M. Morse & Co., Boston: 2500 lbs. wool clips at 13¢, less freight, $24; insurance, ⅛%; commission, 2½%. Find the net proceeds.

5. Account sales of 5000 bushels wheat by the St. Lucas Elevator Co. for account of The Produce and Provision Co., of Milwaukee; Jan. 1, 2000 bu. at $1.05; Jan. 20, 2000 bu. at $1.10; Jan. 30, 1000 bu. at $1.02. Charges: cartage, $25; storage, $12.50; insurance, ¼%; guaranty, ¼%; commission, 2%. Find the net proceeds.

MISCELLANEOUS EXAMPLES.

373. *1.* An attorney collected an account, and after deducting his commission of 7½%, and exchange 75¢ on draft, he remitted his principal $548.70. What was the amount of the collection and how much was the agent's commission?

2. A Hartford fruit dealer sent a Lockport agent $2018.80, and instructed him to buy apples at $1.40 per barrel. If the agent charged 3% for buying, how many barrels did he buy?

3. Find the per cent. of commission on a purchase, if the gross cost is $2048.51, the commission $87.30, the cartage $20, and other charges $1.21.

4. 11500 bushels of wheat were bought through an agent, who charged ⅞% for buying. If the agent paid 85¢ per bushel for the wheat, $162.50 freight, and $12.50 insurance, what sum should be remitted to him in full settlement?

5. A collector obtained 75% of a doubtful account amounting to $1750. What was his per cent. of commission, if, by agreement with the principal, the commission was to be 50% of the net proceeds remitted?

6. A farmer received from his city agent $490 as the net proceeds of a shipment of butter. If the agent's commission is 3%, delivery charges $6.80, and 5% charge is made for guaranty of quality to purchasers, how many pounds, at 27¢ per pound, must have been sold, and how much commission was allowed?

7. Account sales by J. E. Kenning & Co. for acct. Barbon & Boon, New York, sales: Jan. 1, 500 brls. flour, $6.00; Jan. 15, 300 brls. beef, $18. Charges: insurance, ⅛%; commission, 2½%. Find the net proceeds.

8. A Milwaukee agent received $83195.28, with instructions to invest one-half of it in wheat, at 80¢ per bushel, and the balance, less all commissions, in wool, at 20¢ per pound. If his commission for buying the wheat is 2%, and that for buying the wool is 3%, how many pounds of wool will he buy, and what will be the amount of his commissions?

9. Find the net proceeds of a sale made by an agent charging 3½%, if the incidental charges and commission charges were each $41.30.

10. From a consignment of 3160 pounds of tea, sold by an agent at 30¢ per pound, the consignor received as net proceeds $853.74. If the charges for storage and insurance amounted to $51.60, what was the per cent. of commission charged for selling?

11. Birch & Jennings rendered the following account sales to Leroy & Co.: Jan. 1, 300 brls. beef, $22; Jan. 5, 100 brls. pork, $18. Charges: freight, $150; commission, $420; net proceeds, $——. Find rate of commission and net proceeds.

12. Render in full the following account sales, supplying rates per cent. for insurance and commission. Bennett & Ensign for acct. Morton & Blaine: Jan. 1, 1891, 6000# wool, 38¢; 1600 yds. woolen goods, 75¢. Charges: freight, $48; insurance, $4.35; commission, $87; net proceeds, $——.

13. Find the gross proceeds of a sale made by an agent charging 2½% for commission, 5% for guaranty, $17.65 for cartage, $11.40 for storage, and $3.25 for insurance, if the net proceeds remitted amount to $1714.10.

14. A Texas buyer shipped 33000 lbs. of coarse wool to a Boston agent to be sold on commission, and gave instructions for the net proceeds to be invested in leather. If the agent sold the wool at 18¢ per pound, on a commission of 2%, and charged 5% for purchasing and for guaranteeing the grade of the leather, what was the amount of his commissions?

15. A commission merchant received a consignment of 600 bales of cotton, of an average weight of 510 pounds each, which he sold at 12½¢ per pound, on a commission of 3%, charging 10¢ per bale for cartage. He invested for the consignor $9416.20 in bacon, charging 5% for buying, and remitted cash to balance consignor's account. How much was the cash remittance?

BILLS, STATEMENTS AND INVENTORIES.

374. A **Bill** is a written statement in detail of goods sold, or of services rendered.

375. An **Invoice** is a written description of merchandise sold or consigned.

The terms bill and invoice are now used interchangeably. An invoice or bill should give the date of sale, the names of the buyer and seller, the terms of purchase, the items, price, partial products extended of each, and the trade discounts, if any, deducted, the total amount and net amount of bill rendered and manner of delivery or shipment.

376. A **Statement** is an abstract of a personal ledger account, showing under proper dates the detail and total of both debits and credits and the debit or credit balance.

377. An **Inventory** is an itemized schedule of the property possessed by an individual, firm or corporation, and is usually based in value upon the cash standard or what the property is worth at the time of taking inventory.

BILL—STATEMENT.

Colon, June 30, 1891.

William M. Moodey,

To Edin C. White, Dr.

Plumber and Gas Fitter, No. 14 Main St.

Terms: Payable end of month.

1891						
June	1	Balance per bill rendered,			16	50
	5	To 5 hrs. work on gas pipe, 40¢	2	00		
		Fittings 65¢, Iron pipe 70¢, 1 Faucet, $1.25	2	60		
	10	Bath tub $16.00, 8 hrs. work 40¢, $3.20	19	20	23	80
		Total,			40	30
	5	Credit by Cash,			15	00
		Balance due,			25	30

Explanation.—An invoice or bill is rendered on the date of delivery or when the work is done, and in detail. The above is in the form of a statement, as it shows the total debits and credits, and the balance due.

It partakes of the nature of a bill as it details the items since the last statement was rendered on June 1st, when the balance was $16.50. During the month the account has been increased $23.80, making a total to account for of $40.30, on which was paid June 5th cash $15.00, leaving a balance of $25.30.

EXAMPLES.

378. *1.* Find the balance due Jones & Co. from Benj. F. Lee: Jan. 1st, balance rendered, $22.10; 10th, 5 days work, $1.25 per day; lumber and paint, $2.50; 15th, 10 bushels potatoes, $1.20 per bushel. Credit, Jan. 10th, cash $25.00.

2. Geo. W. Banning bought of E. B. Henry & Co., Syracuse, N. Y., June 13, 1891, on account, 2 doz. carpet stretchers, at $3; 10 grindstones, at $2.25; 5 doz. steelyards, at $9; 15 blacksmith drills, at $7; 12 clothes wringers, at $4.50; 6 doz. wrought wrenches, at $12.25; 3 copying presses, at $5; 7 doz. cow bells, at $8.50; 15 doz. cast steel axes, at $12. Find the amount of the bill.

INVOICE.

WORCESTER, Mass., May 15, 1888.

Paulsen & Bro.,
Detroit, Mich.,

Folio 41.
Sales Bk. # 10.

Bo't of Drake & Son,
Wholesale Dry Goods,

Terms :—Cash.

Case.			No. Yd.	Price.	Items.		Amount.	
# 19	15	Pieces Bleached Cotton, 41^2 46^3 41^1 45^2 44 44^1 47^1 45^3 42 42^3 43^3 43^1 47 44 44^2	663^2	6$\frac{3}{4}$¢	44	79		
# 20	12	Pieces Muslin, 37^1 32^3 33 35^3 34^1 32 25^2 33^3 37 38^1 38^1 36	423^3	7¢	29	66	74	45

EXPLANATION.—Under date May 15th the firm of Drake & Son sold Paulsen & Bro. the above bill of goods, terms cash, entered in Sales Book # 10 and on Ledger page 41. In case # 19 there were packed 15 pieces, 663$\frac{2}{4}$ yards bleached cotton which was sold at 6$\frac{3}{4}$¢ per yard, and which amounted to $44.79. In case # 20 were packed 12 pieces muslin, 423$\frac{3}{4}$ yards at 7¢ per yard, amounting to $29.66. The total bill is $74.45. Following each charge are the yards in each piece, as 41$\frac{2}{4}$ yds., 46$\frac{3}{4}$ yds., etc., which are added by horizontal addition and extended into the column headed "No yd."

EXAMPLES.

379. Find the amount of each of the following invoices:

1. Martin & Co. bo't of E. L. Weeks, Jan. 1, 1890, for cash, 10 pc. F. of L. cotton, 50 60^1 65^3 51 60 55 52 62 61 56, at 8¢; 5 doz. C. silk, at 80¢; 4 pc. A. F. cashmere, 62^2 51^3 55 60, at 19¢; 5 pc. A. L. L. cotton, 40 46^2 51^2 55 42^1, at 4¢.

2. Leeland & Bro. sold Gould & Barnes, Jan. 5, 1891, on 30 ds. time: 10 pc. R. gingham, 60 61^2 50^1 60^3 51 61^3 61 50 55 51^3, at 8¢; 10 doz. F. E. braid, at 23¢; 10 pc. B. checks, 45 41 55^1 42 52 40^2 50 55 51^3 45^2, at 24¢; 15 gro. G. buttons, at $\$1.12\frac{1}{2}$; 2 pc. T. A. flannel, 65 60, at 30¢; 6 pc. E. lining, 40 55^1 45^2 52 41 50^1, at 5¢; 5 doz. L. L. gloves, at $3.05.

3. Manderson & Wheeler bought of Brown & Co., Utica, N.Y., Mar. 15, 1891:

10 pc. M. shirting, 40 41 46 34 51 45 50 43 52 42, at 5¢; 15 pc. crash, 600 yd., at 5¢; 6 pc. C. jeans, 50^3 45^1 50 55 51^2 46, at 5¢; 10 doz. M. L. thread, at 59¢; 10 pc. R. print, 41 55 45 51 46 50 40 56 42 52, at $4\frac{1}{2}$¢; 4 pc. N. sateen, 55^3 55 50 60^3, at $5\frac{1}{2}$¢; 5 gross T. braid, at $\$7.62\frac{1}{2}$; 3 doz. L. shirts, at $7.20; 6 pc. T. R. print, 25 35 30^3 31 21 25^1, at $4\frac{3}{4}$¢; 10 cases E. Batts, at $6.00; 20 gro. S. P. buttons, at 49¢; 4 pc. V. barege, 20 23 25 25, at $16\frac{2}{3}$¢; 7 pc. W. print, 45^3 51 45 50 46^2 55 50^3, at $5\frac{1}{4}$¢. Render invoice for same.

BILL SUBJECT TO DISCOUNT.

BOSTON, Mar. 19, 1891.

Messrs. Sanford Bros. & Co.,
Bot. of Lee, Manning & Co.,
Dealers in General Merchandise, etc.
Terms: 30 days, 5% 10 days.

# 625	10	Doz. G. Outseame Gloves,	$15.00	150			
# 610	5	do. L. Und. Kid do.	$22.	110		260	
# 43	11	do. Tin Dippers,	$1.75	19	25		
# 18	2	do. Boilers	$28.	56			
				75	25		
			Less 20%	15	05		
				60	20		
			Less 10%	6	02		
						54	18
						314	18

Explanation.—The number of the goods by which they are called off or ordered are often entered in the margin for convenience in tracing. The first two items, amounting to $260, are sold net. The last two items are discounted 20% and 10%, and the net amount, $54.18, added to the $260 gives a total of $314.18.

380. *1.* Martin Shepherd bought of Gaines & Brown, Jan. 14, 1891, on ℅ 30 days: 500 faucets, $1.50, less 40% and 10%. Find net amount of bill.

2. Charles Sawyer & Bro. sold R. L. Garrity, June 18, 1891, on 60 days, net: 8 doz. pad locks, at $8.00 per doz., less 60%, 25% and 10%. Find the net amount of bill.

3. Booth & Kimball bought of Edwin C. Rodman on ℅, 30 days, 2% 10 days: 5 organs, $120; 10 pianos, $250, less 45% on each; 10 stools, at $1.50 net. Find the net amount of invoice.

4. Farmer & Co. sold Brown & Todd, 30 days net: 2 doz. # 44 dish pans, $2.50 per doz.; 3 doz. # 14 cups, 85¢ per doz.; 6 nests pails, at $1.35 per nest; less discount 25% and 10% from list prices. Find net amount of invoice.

5. McCracken & Post sold W. L. Goodrich on ℅, 60 days, 5% 10 days: 10 tea kettles, 95¢ each; 6 2-quart sauce pans, 65¢ each; 6 3-quart sauce pans, 75¢ each; 2 doz. wash boilers, $30 per doz; ½ doz. # 20 kettles, at $6.00 per doz.; less 40%, 20% and 10% from list prices. Find net amount of invoice.

STATEMENT.

Morristown, May 31, 1891.

Geo. H. Cooper
In acct. with Ogden & Price.
Ledger, 416 "A."

1891. May	1	Account rendered			150	10
	2	Mdse. per bill rendered	36	84		
	5	do. do.	160	16		
	10	do. do.	175	83	372	83
					522	93
		Cr.				
May	2	By cash			100	
		Balance			422	93
		Please remit.				

EXPLANATION.—The above statement is an abstract of Mr. Geo. H. Cooper's account, taken from page 416 of ledger A. On May 1st a statement was rendered him showing a debit balance of $150.10. In this statement the balance of $150.10 is taken as a basis and to it are added the debit items of the ledger, making a total of $522.93. From this total there is deducted a credit found in the ledger of $100 cash, leaving a debit balance of $422.93. When the June statement is rendered this $422.93 will be the first item entered upon it; as, "*June 1, Account rendered, $422.93.*"

Statements are usually rendered each month, at the end of the year, and when called for. A simple statement, without such words as "Please remit" written thereon, is not to be necessarily construed as a request to remit, but an inquiry, such as "Are we correct? We make your acct. $422.93," etc. By such exchange of statements errors are less likely to occur and when made are readily detected.

EXAMPLES.

381. *1.* Elwood Brown's ledger acct. against Neil Bros. & Co. shows the following on Feb. 28th: bal. on Feb. 1st, 1891, $785.50. Debit, 5th, mdse., $500.50; credit, $250; 18th, debit, $88.12; debit, mdse, $18.75; 20th, credit, cash, $150. Find balance due Feb. 28th.

2. Find total amount of debits and the debit balance in a statement from Byron C. Dunning against W. C. Drummond & Co., March 1st: debits, Feb. 18th, mdse., $750; 22d, mdse., $350.50; 25th, mdse., $25.14. Credit, Feb. 20th, cash, $300.

3. Render statement to J. Brown & Co., as shown from your ledger on Jan. 1, 1890, showing balance due: Dec. 5, 1889, debit, mdse, $85.75; Dec. 8, credit cash, $25; 15th, credit mdse., $6.50; same date debit mdse., $150.80.

4. Receive from Pool & Kline a statement of account against yourself, under date May 31st, as follows: Debits, May 5, mdse., $155.14; 10th, mdse., $16.87; 25th, mdse., $110.10. Credit, May 25th, cash, $100; balance, $———. Render statement showing balance.

5. Correct the following statement and show correct footings and balance due: Moore & King, in account with Babcock & Co., Detroit, June 1, 1891: *Debits*, May 1st, mdse., $75.85; 5th, mdse., $100.80; 10th, mdse., $144.15; 31st, mdse., $10.50. Total debits, $330.30. *Credits*, May 10th, cash, $50.00; 15th, cash, $80; 20th, mdse. retd., $16.40; 25th, allowance on damaged goods, $1.54. Total credit, $145.94. Balance due, $184.36.

INVENTORY.

James Hopkins' Inventory, Jan. 1, 1891.

	Schedule A. Merchandise				
5	Car loads Potatoes, 1685 bu. $1.10			1853	50
	Schedule B. Personal Property,				
	3 Horses, $300, $250, $175,	725			
	1 Express Wagon,	120			
	3 Single Harness,	37	50		
				882	50
	Schedule C.				
	Stock groceries per schedule,			1950	75
	Total,			4686	75

Explanation.—Schedules A. and B. are rendered in detail. Schedule C. is given in total. On account of the larger number of items the detail is entered elsewhere. Schedules on loose sheets are compiled by clerks in different departments, and when rechecked and verified the different schedules are entered in a book kept for inventories.

EXAMPLES.

382. Find the inventory in each department in the following examples:

1. Hart & Co's inventory, Jan. 1, 1891. Schedule A. Groceries. 10 brls. flour at $5.50; 5 brls. pork, $16; 1 brl., 38 gals., N. O. molasses, 52¢; 5½ brls., 218 gals., petroleum, 8¢ per gal.; 75 cans peaches, 11¢; 58 cans tomatoes, 8¢; 14 cans Eagle condensed milk, 14¢; 1¼ box Babbitt's soap, $2.40 per box; 145 lbs. powd. sugar, 6¢; 400 lbs. granulated sugar, 4¾¢; 10 boxes matches, 7¢ per box; 300 lbs. Rio, 30¢ per lb.; 110 lbs. Mocha, 38¢; 18 lbs. Java, 36¢; 100 lbs. Oolong, 42¢; 90 lbs. Gunpowder, 40¢; 10 lbs. Eng. Breakfast, 40¢. Total, $——.

2. Schedule B. Furniture and fixtures. 1 office desk, $15.00, less 10%; 1 office safe, $60, less 10%; 1 8-ft. show case, $18.00, less 15%; 2 hanging lamps, $4 and $5 each, net; shelving, $150, less 20%. Total, $——.

3. Schedule C. Ledger accts. J. C. Barnes, folio 18, $83.16; Mary Cameron, folio 20, $44.14; L. B. Moody, folio 18, $14.75; P. W. Brooks, folio 29, $19.98; C. C. Goddard, folio 30, $18.88; Martin W. Cook, folio 42, $162.84; Thos. M. Burch, folio 52, $149.60. Total, $——.

4. Schedule D. Cash, bonds and notes. Cash in safe, $114; in bank, $550; 10 city bonds of $100 each, numbered consecutively 411 to 420 inclusive; 1 note made by Jno. W. Cooper under date Jan. 1, 1888, payable three years after date for $850, with interest, $—; 1 note for $887.50, due March 4th, 1891, with interest to date from Dec. 1, 1890. Total, $——.

5. Schedule E. Real estate. 1 house, and lot # 84 in block # 148 in Pemberton addition, valued at $7000; 1 piece land in section 14, range 28, township 14 of Orleans Co., N. Y., measuring as follows: 185 rods long by 140 rods wide, valued at $125 per acre. Total, $——.

MISCELLANEOUS EXAMPLES.

383. *1.* Darwin E. Goodrich bought of Dence & Bloomingdale, Farmington, May 1, 1891: Terms, 30 days; 10 doz. # 814 knit goods, $4.50; 25 doz. # 855 knit goods, $5.25. Find the amount of invoice.

2. Render the following bill-statement: Brown & Gould "In acct. with" or "To James Osgood, Dr.:" Balance rendered June 1, 1891, $85.13; 5th, 75 lbs. lead pipe, 8¢; 2 nipples, 15¢ each; 2 hours plumber, 40¢; 2 hours helper, 25¢; 10th, cleaning gas fixtures, $2.50; 20th, 1 3″ valve, $4.25; 3 # solder, 25¢ per lb.; credit, June 10th, by cash, $25.00. Balance, $——.

3. Render the following statement of acct.: E. M. Gaylord, acct. on page 325 of Franklin & Potter's ledger, shows the following debits and credits arranged *here* in consecutive dates: Jan. 1, 1891, debit, mdse., per bill, $300.25; 8th, credit, cash, $200; debit, mdse., $62.85; 15th, debit, mdse., $110.82; credit, $100 cash; 30th, debit, mdse., $25.50; credit, cash, $75.00. Balance, $——.

4. Warren & Cowdery, bought of Weeks & Cowles, on 60 ds., 2% 10 ds., Dayton, O., Aug. 15, 1891, goods listed as follows: 5 doz. brass locks, $14.00 per dozen, 25% discount from list; 8 doz. chisels, $3.50 per dozen, 25% and 10% from list; 25 doz. saws, $30 per dozen, 10% from list; 10 doz. hatchets, $5.00 per dozen net. Find net amount of bill to render.

5. Frank Cowles & Son sold James W. McCracken, Colon, Jan. 18, 1891, 30 days, 2% 10 days: 1 case, # 18, 10 pcs. bleached cotton, at 6½¢, per yd., as follows: 41^1, 43, 42^3, 40^2, 44, 39^3, 41^1, 42^2, 43^3, 40^1; 1 case, # 19, 10 pieces blue denims, 22¢, yds. as follows: 45^1, 42, 43^3, 42^2, 44, 43, 42^2, 41^3, 42^1, 44. Find the net amount of invoice.

6. J. M. Long & Co. sold Martin Bros. on ℅. Terms, Oct. 1, 6% 10 ds., 5% 30 ds., 4 mos. net: 100 doz. # 148 gents' buck gloves, lined, $22 per doz.; 50 doz. # 324 gents' buck mitts, $18 per doz. Dated, New York, June 1, 1891. Find the amount of invoice.

Although the date of the bill is June 1, the *terms* make the bill due 4 months from October 1st, or if paid 10 days from October 1st a discount of 6% will be allowed, and if paid 30 days from October 1st a discount of 5% will be allowed.

7. Render the following inventory for Commodore, Ingraham & Co., of Balliston, Vt., Jan. 1, 1891: *Schedule A.* Real estate. House and lot on East Main St. valued at $8500; 10 acres adjacent city property, valued at $500 per acre. *Schedule B.:* personal property. 5 city bonds of $500 each; bills. rec. per bill book, $18.50; ledger accts per ledger B., $650.50. *Schedule C.:* stock of dry goods as per detail, $4850.75.

8. Osgood, Pierpont & Co. sold Jerome & Palmer on ℅, 90 ds., 5% 10 days: 10 pianos, listed at $300 each, less 30% and 10%; 100 family sewing machines, $70 each, less 40%; 18 doz. hammers, $3.00 per dozen, 20%, 10% and 10%; 100 doz. needles, 75¢ per dozen net; 300 doz. assorted thread, 50¢ per dozen net. Dated at Coleman, June 5th, 1891. Find net amount of bill to render.

9. Burt & Simmons bought of Owen, Dale & Co., Boston, March 31, 1891, on ℅ 10 days: 10 boxes glass, $12.50, less 70%, 20% and 10%; 500 # putty at 3¢ per pound net; 100 boxes French plate, $10.75 less 75%, 10% and 5% discount.

10. The Hayden-Brown Manufacturing Co. bought of The New York Chemical Co., June 5, 1891, 30 ds. net: 50 brls. head-light oil, 28¢ per gal.; 44^2, 40^1, 41^3, 43^1, 40^2, 41^2, 42^1, 44^3, 40^2, 41^2, 40^1, 42^2, 43^1, 45^3, 44^2, 41^1, 42^2, 44^3, 43^1, 40^2, 41^3, 40^2, 42^1, 41^3, 40^2, 44^1, 40, 45, 44^3, 40, 41^2, 42, 43^1, 44, 43^1, 41, 42,^{3} 40, 44^3, 45^1, 41^2, 42^1, 44, 45^3, 44, 40, 41^2, 42, 43, 40^1; 10 brls. Newfoundland cod oil, 48¢ per gal.; 40^2, 42, 43^3, 41, 40^1, 43, 42^3, 44^1, 40^2, 45.

MARKING GOODS.

384. To **Mark Goods** is to determine both the cost price and the selling price of the goods, and to represent these prices upon a list or upon tags attached to the articles on sale.

385. The **Cost Price** of an article on sale is the net buying price, or the gross buying price less all discounts.

386. The **Selling Price** of an article on sale is the price asked, or the price asked in connection with the discounts and terms offered.

While the unit of measure varies with the kinds and quantities offered for sale, a large number of manufactured products are sold by the dozen. Jobbers and wholesalers usually buy by the dozen and sell by the dozen. Retailers usually buy by the dozen and sell by the piece.

387. A **Key** is a word, phrase, or an arbitrary arrangement of certain characters, used to represent the nine digits and cipher. The "key" of a business house is used as a private mark to indicate the cost price and selling price to those only who sell the goods.

In choosing a word or words for a key, such selection must be made that no letter is repeated, thus, in such words as handsomely, blacksmith, cash-profit, etc., each one has ten different letters. In the word *handsomely*
1-2-3-4-5-6-7-8-9-0
the key may be determined thus: h-a-n-d-s-o-m-e-l-y; and to indicate the cost price of $3.58 the letters $n. s e. would be the ones to select, for n = 3, s = 5 and e = 8. To avoid the repetition of a letter, any other letter not found in the key may be used as a "repeater."

388. To find the cost price of 1 article, the cost price of 1 dozen being given.

EXAMPLE.—If 1 dozen hats cost $21.00, how much will be the cost of 1 hat?

OPERATION.

$21.00 = cost of 12 hats.
2.10 = $\frac{1}{10}$ of 12 hats.
.35 = $\frac{1}{6}$ of $\frac{1}{10}$.
$1.75 = cost 1 hat.
Or $8\frac{1}{3}$% of $21 = .08$\frac{1}{3}$ × $21 = $1.75.

EXPLANATION.—By cutting off 1 decimal place from the right of dollars $\frac{1}{10}$ of the cost of 12 hats, or $2.10 is obtained. As $\frac{1}{10}$ of the cost of 12 hats = the cost of $\frac{12}{10}$ of 1 hat, and as $\frac{12}{10}$ = $2.10, to find $\frac{10}{10}$ or the cost of 1 hat, diminish $2.10 by $\frac{1}{6}$ of itself, and $2.10 — $.35 = $1.75, or cost of 1 hat. Since 100% represents 1 dozen, $\frac{1}{12}$ of 1 dozen = $8\frac{1}{3}$%; and since 1 dozen cost $21.00, $\frac{1}{12}$ dozen will cost $8\frac{1}{3}$% of $21, or .08$\frac{1}{3}$ × $21.00 = $1.75.

Rule.—*Cut off one figure from the right of dollars in the price by the dozen; the result will be the cost of $\frac{1}{10}$ dozen or $\frac{6}{5}$ of 1 single thing. Diminish this result by $\frac{1}{6}$ of itself and the remainder will be the cost of $\frac{1}{12}$ dozen, or 1 single thing.*

EXAMPLES.

389. Find the cost of 1 article when billed by the dozen as follows:

1. Gloves at $22.50.
2. Hats at $27.00.
3. Ties at $7.50.
4. Boots at $42.
5. Shoes at $19.50.
6. Perfumery at $4.00.
7. Dressing at $5.00.
8. Handkerchiefs at $4.75.
9. Caps at $15.00.
10. Scarfs at $30.00.

390. To find the cost price of 1 article, the cost of 1 dozen being given in dollars and eighths of a dollar.

EXAMPLE.—Find the cost of $\frac{1}{12}$ dozen gloves invoiced at $7.50 per dozen.

In each $\frac{1}{8}$ of a dollar there are $12\frac{1}{2}$¢, or 12 *pence.*

OPERATION.

$$7\frac{1}{2} = cost of 1 doz.
8
———
60 pence = $\frac{1}{12}$ doz.
$2\frac{1}{2}$ ($\frac{1}{3}$ of $7\frac{1}{2}$).
———
$62\frac{1}{2}$¢ = 60 pence.

EXPLANATION.—Since in $1.00 there are 8 *eighths,* in $$7\frac{1}{2}$ there are 60 eighths. Since 1 dozen, or 12 articles, cost 60 eighths, $\frac{1}{12}$ dozen will cost $\frac{1}{12}$ of 60 eighths, or 5 eighths. Since 1 eighth of a dollar = 12 pence, 5 eighths = 5 × 12 pence = 60 pence, cost of $\frac{1}{12}$ dozen; and 60 pence = $62\frac{1}{2}$¢. As the price in pence is found by *multiplying* by *12* and the price of 1 article is found by *dividing* by *12,* therefore the cost of $\frac{1}{12}$ dozen will be as many *pence* as the cost of 1 dozen equals *eights.*

Rule.—*Multiply the cost per dozen by 8, the product will represent the cost in pence of $\frac{1}{12}$ dozen. To this product add $\frac{1}{3}$ of the cost per dozen considered as cents, and the sum will represent the cost in cents.*

This method is much used in some of the New England cities, as it enables one mentally to determine the cost with rapidity; thus, one in marking goods reaches the price $9.00 per dozen and says, 8 × 9 are 72 pence or 75 cents (72 + $\frac{1}{3}$ of 9); at $15 per dozen, 8 × 15 = 120 pence or 125 cents, etc.

EXAMPLES.

391. Find the cost for $\frac{1}{12}$ dozen in pence and then in cents when goods cost as follows:

1. $2.25 per dozen.
2. $3.12½ per dozen.
3. $4.37½ per dozen.
4. $5.50 per dozen.
5. $6.62½ per dozen.
6. $8.75 per dozen.
7. $10.87½ per dozen.
8. $11 per dozen.
9. $13.50 per dozen.
10. $7.50 per dozen.

Call all fractions of ½ cent or more 1 cent extra. For 12½¢ or any multiple of 12½¢ read eighths.

392. To find the selling price of 1 article, when the cost price for 1 dozen and the rate per cent. of profit are given.

EXAMPLE.—Find the selling price for 1 pr. shoes so as to net a profit of 33⅓%, the cost per dozen being $19.50.

OPERATION.

1 dozen = cost, $19.50
Profit = ⅓ = 6.50
Selling price = $26.00
$\frac{1}{10}$ of $26.00 = 2.60
Less ⅙ = 43
Net cost $\frac{1}{12}$ doz. = $2.17
Or 8⅓% of $26.00 = $2.16⅔
in business, $2.17.

EXPLANATION.—Since 33⅓% profit = ⅓ of the cost, ⅓ of the cost added to $19.50 = $26, the selling price for 1 dozen. Given the cost of 12 articles, find the cost of 1 article by Art. 388.

Rule.—***Find the selling price of 1 dozen by adding to the cost of 1 dozen such a part of the cost as the rate per cent. profit is a part of 100%. Find the selling price of 1 article by taking 8⅓% of the price for 1 dozen.***

Observe that $\frac{1}{10}$ of the cost price will give the cost of 1 article at a profit of 20%; as, cost $15.00 per doz. = $1.50 for $\frac{1}{12}$ doz. at a profit of 20%.

EXAMPLES.

393. Find the selling price for 1 article on goods costing as follows to realize accompanying gains:

1. Cost $15, gain 25%.
2. Cost $9.00, gain 33⅓%.
3. Cost $13.50, gain 6¼%.
4. Cost $18, gain 16⅔%.
5. Cost $21.00, gain 20%.
6. Cost $7.50, gain 33⅓%.
7. Cost $3.75, gain 25%.
8. Cost $27.50, gain 33⅓%.
9. Cost $19.50, gain 16⅔%.
10. Cost $25.50, gain 20%.

394. To find the selling price at which goods must be marked to insure a given per cent. profit and allow a trade discount to the purchaser.

EXAMPLE.—A manufacturer finds that certain novelty goods cost him $10.50 per dozen, packed. At what list price per dozen must he mark them, to allow a trade discount of 30% and still make a profit of 33⅓% on the cost.

OPERATION.

Cost = $10.50
Profit = ⅓ of 10.50 = 3.50
Net price = $14.00
List price = 100%
Rate disct. = 30%
Net price = 70%
As 70% = $14.00, net price
100% = $20.00, list price.

EXPLANATION.—Since the cost is $10.50 and the profit is 33⅓% of the cost, therefore, the net price = $10.50 + $3.50 = $14.00. Since the selling or list price may be represented by 100% and the rate of discount is 30%, therefore, 100% — 30% or 70% of the list = the net price or $14. As 70% = $14,00, 1% = $14 + 70 = $ .20 and 100% = 100 × $.20 = $20.00. Proof: 30% of $20 = $6.00 and $20 — $6 = $14 net.

Rule.—*Add to the cost the required gain, and divide the list price thus obtained by the net per cent. of sale.*

EXAMPLES.

395. *1.* At what price shall goods be marked which cost $16.80 so as to be discounted 25% and still net a profit of 25%.

2. At what price shall the following goods be marked per dozen so as to allow a discount of 40% and 10% and still make a profit of 33⅓% on the cost? Hats at $8.10, $4.05, $16.20 and $24.30 per doz.; gloves at $20.25, $10.12½ and $30.37½ per doz.

3. What shall be the marking price of the following goods so as to net a profit of 25% after allowing a discount of 20%? Hats per dozen, $18.00, $12.00, $16.00; ties, per doz., $6, $12, $18; gloves, per doz., $10, $15, $20.

396. To read the cost price of goods marked in letters and to determine the selling price therefrom and to mark same in letters.

EXAMPLE.—A salesman is required to mark the selling price of a stock of goods for a firm whose key word is "HANDSOMELY," and whose repeater is "R." The cost price is already marked upon the goods. What shall be the selling price in dollars and cents of 1 doz. ties to make a profit of 20%, the cost per dozen being marked $N. S Y?

OPERATION.

Cost $N. S Y. = $3.50

Sell $D. A Y. = $4.20

EXPLANATION. — Since the letters N. S Y. represent the 3d, 5th and 10th letters of the key word, therefore, for N. write 3 for S. write 5 and for Y. write 0, and hence, the cost price = $3.50. To determine the selling price of 20% on cost add ⅕ of cost to itself or $3.50 + 70¢ = $4.20, the selling price. For $4.20 written in the key word select the 4th, 2nd and 10th letters respectively and write $D. A Y.

Rule—I. *To read prices marked in the letters of a key: Read for each letter the figure indicated by the letter according to its order or position numerically in the key.*

II. *To mark prices in letters by the key: Write for each figure the letter indicated by its order or position numerically in the key.*

EXAMPLES.

397. Read, or write in figures the cost price of the following articles and mark the selling price at an advance of 33⅓% over cost in the key word "Handsomely;" repeater, any other letter.

1. Gloves, $M. S Y. per doz.
2. Shoes $N O. Y X. per doz.
3. Hats, $D E. S Y. per doz.
4. Ties, $N. Y G. per doz.
5. Collars, $D. S Y. per doz.
6. Caps, $L. S Y. per doz.
7. Knives, $N. M S. per doz.
8. Spoons, $O. S Y. per doz.
9. Forks, $ N. S Y. per doz.
10. Shoes, $A S. C Y. per doz.
11. Books, each, $H. Y X.
12. Paper per qr. $. A Y.
13. Berries per crate, $A. D Y.
14. Potatoes per bu., $H. X Y.
15. Flour per brl., $S. M S.
16. Sugar per lb., D½¢.
17. Coffee per lb., N M½¢.
18. Starch per lb., M½¢.
19. Oranges per box, $S. L Y.
20. Lemons per box, $O. M S.

MISCELLANEOUS EXAMPLES.

398. *1.* If 1 doz. ties cost $5.50, what is the cost price of 1 tie?

2. George Irwin bought of Thos. L. King & Co., 10 doz. steam valves, $30 per doz.; 5 doz. lamps at $7.50 per doz.; and 4 doz. files at $5 per doz.; less 40% discount from the list. Find the net cost of each article.

3. Find the selling price of 1 hat, which cost by the dozen $9.00, so as to make a profit of 20%.

4. Burnside, Banks & Co. bought of Clarkson & Loomis, 15 doz. imported fur hats at $80 per doz., less 30%, 25% and 10% from list. Find the net cost of 1 hat.

5. In the key "black horse" mark the cost price of 1 knife bought at $4.00 per dozen and the selling price at 25% advance.

6. The Inventory Book of the Columbia Gas Co. showed on Jan. 1, 1891, the following articles invoiced by the dozen: Washers, $2.25; steam caps, $25.50; files, $4.62½; chimnies, 90¢; rules, $1.80; sinks, $56.00. Find the cost of 1 article of each kind.

7. Give the cost price of 1 of each of the following articles bought by the dozen: Ink, costing 36¢; ink wells, $1.25; slates, $1.50; skates, $3.00; sleds, $15.00.

8. The Goodale Rubber Co. buy goods listed per doz. as follows: Rubber boots, $54.00, $39 and $32; coats, $86.00, $74, $63 and $57. Find the cost of 1 article or pair of each kind.

9. Write in figures the following cost prices marked in the key word "blacksmith," repeater X: Plates, $A. K H; bowls, $B. L K; platters, $T. H X; goblets, $B. K H; tea sets, $B L. H X; dinner sets, $A H. K H; breakfast sets, $B K. H X.

10. The Scott-Joice Glove Co. imported hosiery and knit goods at a net cost per dozen as follows: Hosiery, $2.12½, $2.37½, $2.50, $2.62½, $2.75, $3.00, $3.50 and $4.00; knit goods, $7.50, $8.00, $8.12½, $8.50, $9.87½, $10.00, $10.50 and $11.00. Find the cost price of 1 article or pair of each kind.

INTEREST.

399. **Interest** is the price paid for the use of money.

400. The **Principal** is the money for the use of which interest is paid.

401. The **Amount** is the sum of the principal and interest.

402. The **Time** is the period during which the principal bears interest.

403. The **Rate of Interest** is the annual rate per cent.

Rate per cent. per annum means that a certain number of *cents* is paid for the use of *one* dollar for *one* year.

Interest is reckoned as a certain per cent. of the principal. It is therefore a Per Cent. of which the Base is the principal.

404. Interest differs from the preceding applications of Percentage only in the introduction of time as an element in connection with the rate per cent.

The Principal = the Base, the Rate Per Cent. = the Rate, the Interest = the Percentage, the sum of the Principal and Interest = the Amount.

405. **Legal Interest** is the rate fixed by law.

406. **Usury** is interest taken at a *higher* rate than the law allows.

407. **Simple Interest** is interest on the principal only, for the whole time of the loan or credit; and this is generally understood by the term *interest.*

408. **Annual, Semi-Annual,** or other **Periodic Interest,** is interest computed at a specified rate for a year, half-year or other designated period.

409. **Compound Interest** is interest computed on the amount at regular intervals.

The payment of *periodic* interest, if specified in a contract, may usually be enforced; and if not made when due, it becomes simple interest bearing, and is not usury.

Neither the paying nor the receiving of compound interest is usury; but its payment cannot ordinarily be enforced, even though it is mentioned in the contract.

410. Common Interest is interest computed on a basis of 360 days for a year.

This method is generally employed by business men, and in some States has received the sanction of law.

In reckoning interest by this method, it is customary to consider a year to be 12 months, and a month to be 30 days.

In computing interest for short periods of time, it is customary to take the exact number of days.

When no particular rate of interest is named in a contract containing a general interest clause, as "with interest," or "with use," the legal rate of the place where the contract is made is understood.

Debts of all kinds bear interest *after* they become due, but not *before*, unless specified.

411. Exact Interest is interest computed for the exact time in days, and regarding the days as 365ths of a year. This method is used by the United States Government and by some merchants and bankers; but as it is inconvenient unless interest tables are used, it is not generally adopted.

Exact interest, for any period of time expressed in days, may be obtained by subtracting $\frac{1}{73}$ part from the common interest for that period of time.

Common interest may be obtained from exact interest by adding thereto $\frac{1}{72}$ part of itself.

SIX PER CENT. METHOD.

412. The following method of computing interest is based upon time as usually reckoned; *i. e.*, 12 months of 30 days each, or 360 days for a year, and is called the *Six Per Cent. Method.* It is convenient for use in all cases where time is not given in days, as for years and months, or for years, months, and days, and where exact interest is not required. Should the rate be other than six per cent., the change can be easily made. It is a common method of computing interest.

TABLE.

$1.00 in 1 yr., at 6%, will produce $.06 interest.
$1.00 in $\frac{1}{6}$ yr., or 2 mo., at 6%, will produce $.01 interest.
$1.00 in 1 mo., or 30 da., at 6%, will produce $.005 interest.
$1.00 in 6 da., or $\frac{1}{5}$ mo., at 6%, will produce $.001 interest.
$1.00 in 1 da., at 6%, will produce $.000$\frac{1}{6}$ interest.

413. To find the interest on $1.00 for years, months and days, at 6% per annum.

EXAMPLE.—Find the interest on $1.00 for 1 year, 5 months and 13 days.

OPERATION.

1 yr. 5 mos. = 17 mos.
16 mos. = $.08.
1 mo. = .005.
12 ds. = .002.
1 da. = .000$\frac{1}{6}$.
17 mos. 13 ds. = $.087$\frac{1}{6}$.

EXPLANATION.—Since the interest for 1 month = ½¢, for 1 year and 5 months, or 17 months, the interest will be .085. Since the interest for 1 month = ½¢ or 5 mills, for 1 day the interest is $\frac{1}{30}$ or $\frac{1}{6}$ mill, and for each 6 days it will be 1 mill.

Rules.—1. *Take as many cents as equal ½ the number of months, 1 mill for each six days and ⅙ of a mill for each remaining day. Or,*

2. *Divide the months by 2 and the days by 6. The quotient will be cents for the first and mills for the second. The sum of the partial quotients will be the interest on $1.00 for the given time at 6%.*

EXAMPLES.

414. Find the interest on $1.00 at 6% for

1. 1 yr., 7 mos., 24 ds.
2. 1 yr., 8 mos., 12 ds.
3. 3 yrs., 5 mos., 18 ds.
4. 1 yr., 9 mos., 21 ds.
5. 1 yr., 10 mos., 27 ds.
6. 1 yr., 1 mo., 1 da.
7. 2 yrs., 3 mos., 7 ds.
8. 1 yr., 8 mos., 22 ds.
9. 3 yrs., 10 mos.
10. 1 yr., 29 ds.
11. 1 yr., 11 mos., 1 da.
12. 1 yr., 1 mo., 2 ds.
13. 2 yrs., 2 mos., 3 ds.
14. 1 yr., 3 mos., 4 ds.
15. 2 yrs., 4 mos., 5 ds.
16. 3 yrs., 5 mos., 7 ds.
17. 2 yrs., 5 mos., 8 ds.
18. 1 yr., 7 mos., 10 ds.
19. 2 yrs., 9 mos., 11 ds.
20. 1 yr., 11 mos., 13 ds.

415. To find the interest on any principal for any time at 6% by the Six Per Cent. Method.

EXAMPLE.—Find the interest on $540 for 1 yr., 7 mos., 21 ds., at 6% per annum.

OPERATION.

Interest on $1. for
19 mos. = $.095.
21 ds. = .0035.
Total, $.0985.
Interest on $540 =
540 × $.0985 = $53.19.

EXPLANATION.—The interest on $1 for 1 year and 7 months = $ .095. The interest for 21 days = $.0035, and $.095 + $.0035 = $.0985, the interest on $1. Since the interest on $1 for the time = $.0985, the interest on $540 = 540 × $.0985 = $53.19.

Rule.—*Multiply the interest on $1.00 for the given time at 6% by the principal, the product will be the interest on the principal.*

EXAMPLES.

416. Find the interest at 6% on

1. $650 for 1 yr. 4 mo. 12 ds.
2. $750 for 6 mos. 8 days.
3. $475 for 18 mos. 27 days.
4. $955 for 1 yr. 1 mo. 1 da.
5. $857 for 1 yr. 6 mos. 12 ds.
6. $3500 for 6 mos. 15 days.
7. $4000 for 17 mos. 13 days.
8. $5500 for 3 yrs. 6 months.
9. $8500 for 1 yr. 10 months.
10. $73800 for 4 mos. 18 days.
11. $5000 for 1 mo. 3 days.
12. $6000 for 2 mos. 3 days.
13. $7500 for 3 mos. 3 days.
14. $9500 for 4 mos. 3 days.
15. $500 for 33 days.
16. $800 for 63 days.
17. $750 for 93 days.
18. $980 for 123 days.
19. $5415612.50 for 1 day.
20. $1850.75 for 1 yr. 1 mo. 1 da.

As the interest on $1.00 for a given time equals cents, or dollars and cents, therefore on any number of dollars it is *times* the interest on $1.00. Hence, where the principal has cents write the principal as an abstract number with cents as decimals.

Where the time is expressed in days more than 1 month, proceed as usual to divide by six; thus, 123 days ÷ 6 = 20½, or 20½ mills, which equal $ .0205.

417. To find the interest on any principal for any time at any rate per cent.

EXAMPLE.—Find the interest on $6000 for 1 yr., 4 mos. and 13 ds. at 8% per annum.

OPERATION.

Interest on $1.00 = $ .082⅙.
Interest on $6000. for 16 mos. 13 ds. =
6000 × $ .082⅙ = $493.00 at 6%.
2% = ⅓ of 6% and ⅓ = 164.33 at 2%.
Total int. 16 mos. 13 ds. = $657.33 at 8%.

EXPLANATION.—Since the interest on $1. for the given time equals $ .082⅙, on $6000 the interest will equal 6000 times the interest on $1, or $493. Since 8% is ⅓ more than 6%, therefore the interest at 8% will be ⅓ more than the interest at 6%, and ⅓ of $493 = $164.33, which added = $493.00 + $164.33 = $657.33, the interest on $6000 at 8% for 1 year, 4 months and 13 days.

Rule.—*Find the interest on the given principal for the given time at 6%. Then add to or take from the interest thus obtained such a part of itself as the difference between 6% and the given rate per cent. is a part of 6%.*

To find the interest at 7% add $\frac{1}{6}$; 7½%, add $\frac{1}{4}$; 8%, add $\frac{1}{3}$; 9%, add $\frac{1}{2}$; 10%, divide by 6 and remove the decimal point 1 place to the right; at 5%, subtract $\frac{1}{6}$; at 4½%, subtract $\frac{1}{4}$; at 4%, subtract $\frac{1}{3}$; at 3%, take $\frac{1}{2}$; 2%, take $\frac{1}{3}$; 1%, take $\frac{1}{6}$, using in each case the interest at 6% as a basis.

EXAMPLES.

418. Find the interest on

1. $450 for 1 year, 7 months, 15 days, at 7%.
2. $700 for 2 years, 3 months, 12 days, at 7½%.
3. $800 for 1 year, 1 month, 19 days, at 8%.
4. $1200 for 18 months, 15 days, at 10%.
5. $1600 for 1 year, 33 days, at 9%.
6. $4500 for 10 months, 27 days, at 5%.
7. $17545 for 2 years, 1 month, 10 days, at 4%.
8. $1305 for 1 year, 1 month, 7 days, at 6%.
9. $1743.50 for 93 days, at 4½%.
10. $2100.50 for 16 months, 1 day, at 6½%.

BANKERS' METHODS.

419. To find the interest on any principal for any number of days, at any rate per cent., by the Bankers' 60-day method.

EXAMPLE.—Find the interest on $3600 for 76 days at 8%.

OPERATION.

$36	00	= Int. 60 ds.
9	00	= Int. 15 ds.
	60	= Int. 1 da.
$45	60	= Int. 76 ds. at 6%
15	20	= Int. 76 ds. at 2%
$60	80	= Int. 76 ds. at 8%

EXPLANATION.—Since the interest for 60 days = $\frac{1}{100}$ or .01 of the principal, the interest on $3600 is found by cutting off *two* places from the right of dollars to be $36.00. The interest for 15 days (the next highest aliquot, see Article 113, page 37) = $\frac{1}{4}$ of $36, or $9. The interest for the 1 remaining day = $\frac{1}{6}$ of the interest for 6 days ($3.60) = 60¢, and $36 + $9 + 60¢ = $45.60, the interest on $3600 for 76 days at 6%. As 8% exceeds 6% by 2% and 2% being $\frac{1}{3}$ of 6% add to $45.60, $\frac{1}{3}$ of itself; and $45.60 + $15.20 = $60.80.

Rule.—I. *To find the interest on any principal at 6% for 60 days, cut off two places from the right of dollars; for 6 days cut off 3 places.*

II. *For any number of days take such a part of the interest for 60 days or for 6 days as the given number of days is a part, more or less, of 60 days or of 6 days.*

EXAMPLES.

420. Find the interest on

1. $800 for 15 days at 6%.
2. $900 for 48 days at 7%.
3. $500 for 6 days at 9%.
4. $750 for 36 days at 10%.
5. $1200 for 54 days at 7½%.
6. $1400 for 24 days at 7%.
7. $150 for 42 days at 5%.
8. $450 for 7 days at 8%.
9. $500 for 1 day at 7%.
10. $1800 for 3 days at 9%.
11. $250 for 10 days at 6%.
12. $870 for 12 days at 8%.
13. $951 for 30 days at 7½%.
14. $1770 for 20 days at 4½%.
15. $3300 for 40 days at 4%.
16. $751.50 for 67 days at 8%.
17. $1475.10 for 61 days at 7%.
18. $1860 for 73 days at 8%.
19. $1250 for 96 days at 7½%.
20. $5000 for 123 days at 10%.

The rules governing interest by the United States Money Method will apply in this part of the subject, by substituting *hundredths* and *thousandths* of a dollar for *cents* and *mills*. See pages 36 to 43.

421. To find the interest on any principal for years, months and days at any rate per cent. by the Bankers' 30-day method.

The foregoing method in which the interest for 2 months or 60 days is used as a basis or standard of comparison, is valuable for all short time calculations. For periods of years, months and days the one month basis is used.

EXAMPLE.—Find the interest on $800 for 1 year 3 months and 27 days at 8%.

OPERATION.

Int. on $800 for 2 mos. = $8|00
Int. on $800 for 1 mo. = $4|00
1 yr. 3 mos. 27 ds. = 15.9 mos.
15.9 × $4 = $63.60 Int. at 6%
⅓ of $63.60 = 21.20 Int. at 2%
Total, $84.80 = Int. at 8%

EXPLANATION.—One half of the principal with two decimal places cut off from the right of dollars equals the interest for 1 mo., and ½ of $8.00 = $4. 1 year 3 months and 27 days equal 15$\frac{9}{10}$ months or 15.9 months; and as 1 month's interest = $4, the interest for 15.9 months will be 15.9 times $4; and 15.9 × $4 = $63.60, the interest at 6%. The interest at 8% is ⅓ more than the interest at 6%, and ⅓ of $63.60 = $21.20; $63.60 + $21.20 = $84.80, int. at 8%.

Rule.—I. *Cut off two places from the right of dollars and ½ of the result will be the interest for 1 month at 6%.*

II. *Change the years and months to months and add to the total of months ⅓ the number of days as tenths of a month. Their sum will be months and the decimal of a month.*

III. ***Multiply the interest for 1 month, thus obtained, by the number of months, and the product will be the interest for the given time at 6%.***

The interest at any rate per cent. for 1 month equals so many *twelfths* of each 1 *hundredth* of the principal; therefore 4%, 5%, 6%, 7%, 8%, 9% and 10% are respectively $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{2}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ *hundredths* of the principal. It will be seen then that, *after cutting off two places* inthe *principal*, to obtain the interest for 4%, it is only necessary to take $\frac{1}{3}$; for 6%, $\frac{1}{2}$; for 8%, $\frac{2}{3}$ of the principal. For 5% take $\frac{1}{6}$ *less* than $\frac{1}{2}$; for $\frac{7}{12}$, $\frac{1}{6}$ *more* than $\frac{1}{2}$; for 10%, $\frac{1}{6}$ less than the principal.

As 21 days, 18 days or 3 days, equal respectively so many 30ths of a month, to change $\frac{21}{30}$, $\frac{18}{30}$ or $\frac{3}{30}$ to a decimal, the numerator and denominator must be divided by 3; and $\frac{7}{10}$, $\frac{6}{10}$ and $\frac{1}{10}$ express 21 days, 18 days and 3 days as decimals of a month. Dividing the days by 3, then and writing the quotient after the decimal point, changes the days to decimals of a month. Days not divisible by three will have $\frac{1}{3}$ or $\frac{2}{3}$ tenths remaining; as 1 day, 5 days, 7 days, etc. = .$\frac{1}{3}$, .1$\frac{2}{3}$, .2$\frac{1}{3}$, etc.

EXAMPLES.

422. Find the interest for *1 month* at 6% on

1. $8000.
2. $1740.
3. $960.
4. $750.
5. $814.
6. $2840.
7. $1670.
8. $1450.
9. $2812.
10. $2628.
11. $1450.60.
12. $755.90.
13. $1275.84.
14. $28.50.
15. $78.54.

EXAMPLES.

423. Change the following years, months and days to months and decimals of a month.

1. 1 yr. 4 mos. 3 ds.
2. 1 yr. 3 mos. 6 ds.
3. 1 yr. 5 mos. 9 ds.
4. 1 yr. 10 mos. 12 ds.
5. 1 yr. 8 mos. 15 ds.
6. 1 yr. 1 mo. 18 ds.
7. 1 yr. 11 mos. 21 ds.
8. 2 yrs. 1 mo. 24 ds.
9. 3 yrs. 3 mos. 27 ds.
10. 3 yrs. 5 mos. 1 da.
11. 2 yrs. 3 mos. 4 ds.
12. 3 yrs. 10 mos. 5 ds.
13. 1 yr. 9 mos. 7 ds.
14. 1 yr. 11 mos. 8 ds.
15. 1 yr. 6 mos. 10 ds.
16. 1 yr. 4 mos. 11 ds.
17. 1 yr. 9 mos. 29 ds.
18. 3 yrs. 10 mos. 28 ds.
19. 1 yr. 1 mo. 1 da.
20. 1 yr. 11 mos. 28 ds.

EXAMPLES.

424. Find the interest on the following by taking $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$ or $\frac{3}{4}$ *of*, or by subtracting $\frac{1}{6}$ from, one-half or the whole of the principal, as indicated by the rate as so many *hundredths*:

1. On $900 for 1 yr. 11 mos. 28 days at 8%.
2. On $750 for 1 yr. 4 mos. 21 days at 4%.
3. On $6000 for 2 yrs. 10 mos. 18 days at 10%.
4. On $5000 for 1 yr. 8 mos. 27 days at 9%.
5. On $3672 for 1 yr. 4 mos. 22 days at 5%.
6. On $7554.50 for 1 yr. 2 mos. 23 days at 6%.
7. On $1860.72 for 1 yr. 11 mos. 17 days at 7%.
8. On $2460 for 1 yr. 10 mos. 25 days at 4½%.
9. On $1624 for 1 yr. 2 mos. 3 days at 7½%.
10. On $3472.12 for 4 yrs. 10 mos. 27 days at 10%.
11. On $3200 for 3 yrs. 8 mos. 27 ds. at 8%.
12. On $1512.75 for 1 yr. 1 mo. at 7½%.
13. On $500 for 3 yrs. 2 mos. 12 ds. at 7%.
14. On $1275 for 1 yr. 9 mos. 18 ds. at 9%.
15. On $1550 for 3 yr. 10 mos. 22 ds. at 5%.

For 4½% subtract ¼, and for 7½% add ¼, of the int. at 6%.

425. To find the principal, the rate, interest and time being given.

As the elements of interest are the base, rate, percentage and time, or the principal, rate, interest and time, if any three of these are given the fourth may be found.

EXAMPLE.—What principal at 6% for 1 yr. 2 mos. 12 ds. will produce $57.60 interest?

OPERATION.

100% = principal.
6% = rate.
$57.60 = interest.
$ 1.00 = unit of meas.
14 mos. 12 ds. = time.
Int. on $1.00 = $.072.
$57.60 ÷ $.072 = 800.
800 × $1.00 = $800, or princp.

EXPLANATION. — The elements given are the rate, interest and time. The element wanted is the principal. The unit of measure is $1.00. Since the interest on $1.00 for the given time = $.072, it will take as many times $1.00 to produce $57.60 as $.072, is contained times in the given interest, $57.60; and $57.60 ÷ $.072 = 800; and 800 × $1.00 = $800, the principal.

Rule.—*Divide the given interest by the interest on $1.00 for the given time at the given rate per cent., the quotient will be times $1.00 of the principal.*

EXAMPLES.

426. What principals at 6% will produce interest as follows:

1. $72 in 1 yr. 4 months,
2. $71.10 in 1 yr. 10 mos. 18 ds.
3. $92.40 in 1 yr. 8 mos. 16 ds.
4. $131 in 2 yrs. 1 mo. 24 ds.
5. $68.20 in 1 mo. 12 ds.
6. $6.60 in 33 days ?
7. $12.60 in 63 days ?
8. $21.70 in 93 days ?
9. $36.90 in 123 days ?
10. $.82 in 3 days ?

427. To find the rate, the principal, interest and time being given.

Example.—At what rate per cent. will $120 produce $11.20 interest in 1 yr. 4 mos.?

OPERATION.

100% = principal = $120.
$11.20 = interest.
1% = unit of measure.
Int. on $120 at 6% = $9.60.
1% int. = $9.60 ÷ 6 = $1.60
$11.20 ÷ $1.60 = 7 times.
7 × 1% = 7% = rate.

Explanation.—The elements given are the principal, interest and time. The element wanted is the rate, the unit of measure being 1%. The interest on $120 at 1% for the time is ⅙ of 6% or ⅙ of $11.20 = $1.60. As the interest at 1% = $1.60, the given interest will represent as many per cent. as $1.60 is contained *times* in $11.20; and $11.20 ÷ $1.60 = 7 times, or 7%, the required rate.

Rule.—*Divide the given interest by the interest on the principal at 1% for the given time, the quotient will be the number of times the rate of 1%.*

To find 1% interest, the shortest way is to take ⅙ of the interest at 6% for the time.

EXAMPLES.

428. At what rates per cent. will the following principals produce the given interest for the given time :

1. $800 produce $64 in 16 mos ?
2. $750 produce $61.87½ in 1 yr. 4 mos. 15 days ?
3. $940 produce $80.84 in 1 yr. 27 days ?
4. $5000 produce $13.75 in 33 days ?
5. $12000 produce $77.50 in 93 days ?
6. $960 produce $7.68 in 48 days ?
7. $542 produce $2.71 in 30 days ?
8. $624 produce $10.19 + in 98 days ?
9. $5000 produce $131.25 in 105 days ?
10. $3500 produce $5.25 in 6 days ?

429. To find the time, the principal, interest and rate being given.

EXAMPLE.—In what time will $900, at 7% produce $76.65 interest ?

OPERATION.

\$900 = principal.
\$ 76.65 = interest.
7% = rate.
1 yr. = unit meas.
Int. 1 yr. on \$900 = \$63.
$\$76.65 \div \$63 = 1.21\frac{2}{3}$, or $1.21\frac{2}{3}$ yrs.
$.21\frac{2}{3}$ yrs. = $.21\frac{2}{3}$ of 12 mos. = 2.6 mos.
.6 mos. = .6 of 30 ds. = 18 ds.
Time = 1 yr. 2 mos. 18 days.

EXPLANATION.—Since the principal, interest and rate are given to find the time, and the unit of measure is 1 year, it is evident that it will take as many years to produce \$76.65 as the interest on \$900 for 1 yr., at 7%, or \$63, is contained *times* in \$76.65; and $\$76.65 \div \$63 = 1\frac{21\frac{2}{3}}{100}$ times. Reducing $1.21\frac{2}{3}$ yrs. to integers, the time = 1 yr., 2 mos., 18 days.

The unit of measure may be 1 *year*, 1 *month* or 1 *day*, according as the time appears short or long. To illustrate : The interest on the above \$900 for 1 mo. at 7% = \$5.25 and 76.65 ÷ \$5.25 = 14.6 or 14.6 mos. = 14 mos. 18 ds., or 1 yr., 2 mos., 18 days.

Rule.—*Divide the given interest by the interest on the principal at the given rate per cent. for 1 year, 1 month or 1 day. The quotient will indicate the time in years, months or days, according as the unit of measure is years, months or days.*

EXAMPLES.

430. Find the time in which the following principals will produce the given interest at the given rates per cent.

1. \$600, int. \$21 at 7%.
2. \$860, int. \$120.40 at 7%.
3. \$1200, int. \$6.30 at 9%.
4. \$5000, int. \$31.25 at $2\frac{1}{2}$%.
5. \$9000, int. \$18.75 at 5%.
6. \$1500, int. \$1.25 at 5%.
7. \$8000, int. \$90 at 9%.
8. \$1250, int. \$75 at 6%.
9. \$950, int. \$251.75 at 10%.
10. \$25000, int. $\$416.66\frac{2}{3}$ at 6%.

431. To find the principal, the amount, rate per cent. and time being given.

EXAMPLE.—What principal will amount to \$321.75 in 1 year 2 months and 15 days at 6% ?

OPERATION.

100% = principal.
6% = rate.
106% = amt. 1 yr.
14 mos. 15 ds. = time.
107¼% = $321.75 for 14½ mo.
1%. = $3.
100% = $300, principal.

EXPLANATION. — Since 100% represents the amount for 1 year, therefore for 14 mos. and 15 ds. the amount per cent. = 107¼% which equals, in dollars, $321.75, the given amount. As 107¼% = $321.75, 1% = $321.75 ÷ 107.25 = $3; and 100% = 100 × $3 = $300. Proof: The interest on $300 for 1 month = $1.50 (Art. 415) and for 14 mos. 15 ds. it will be 14.5 × $1.50 = $21.75 interest. The amount will equal $300 + $21.75 = $321.75.

Rules.—1. ***Divide the given amount by the amount per cent. for the given time, the quotient will represent the principal. Or,***

2. ***Divide the given amount by the amount of $1.00 for the given time at the given rate per cent.***

EXAMPLES.

432. What principal will amount to

1. $354, in 3 yrs. at 6%?
2. $552.50, in 1 yr. 6 mo. at 7%?
3. $872, in 2 yrs. 3 mos. at 4%?
4. $713.50, in 1 yr. 4 mo. at 8%?
5. $345., in 1 yr. 6 mo. at 10%?
6. $402.20, in 33 ds. at 6%?
7. $858.92½, in 63 ds. at 6%?
8. $50.27½, in 33 ds. at 6%?
9. $153.10, in 93 ds. at 8%?
10. $15001.25, in 1 day at 3%?

EXACT INTEREST.

433. To find the interest on any principal at 6%, on a basis of 365 days to the year.

Aside from uses in government calculations, *exact* interest is rarely computed; and while it is *enforceable*, being *strictly legal*, the greater convenience of the 360-day rules so commend them to public favor as to lead to their common use.

On a basis of 12 periods of 30 days each, or 360 days for a year, the year's interest is taken for a period too short, since the year (exclusive of leap year) contains 365 days. The time is, therefore, 5 days or $\frac{5}{365}$, equal to $\frac{1}{73}$, too short, and the interest taken on that basis is proportionally too great; to correct this error and obtain the *exact* interest, subtract $\frac{1}{73}$ part from any interest obtained on a 360-day basis.

EXAMPLE.—Find the exact interest on $630 for 50 days at 6%.

OPERATION.

Interest for 60 ds. =	$6	30
Int. for 10 ds. = $\frac{1}{6}$ =	1	05
Interest for 50 ds. =	$5	25
Less $\frac{1}{73}$ =		07
Exact interest =	$5	18

EXPLANATION.—The ordinary interest on a basis of 360 days is $5.25. Since the ordinary interest is $\frac{1}{73}$ more than the exact interest, therefore, by subtracting from $5.25, $\frac{1}{73}$ part of itself, gives $5.18, the exact interest.

Rule.—*Find the interest on the given principal for the given time at the given rate per cent. on a basis of 360 days, and subtract $\frac{1}{72}$ part of the interest thus obtained. The difference will be the exact interest for 365 days.*

To change *exact* interest to ordinary interest add $\frac{1}{72}$ part of itself.

EXAMPLES.

434. *1.* Find the *exact* interest of $954, for 63 days, at 7%.

2. Find the *exact* interest on $800, for 33 days, at 5%.

3. Find the *exact* interest on $137.50, for 93 days, at 8%.

4. Find the *exact* interest on $210.54, for 100 days, at 9%.

5. Find the *exact* interest on $681.80, for 90 days, at 10%.

6. Find the *exact* interest on $500, for 48 days, at 6%.

7. Find the *exact* interest on $1200, for 31 days, at 5%.

8. Find the *exact* interest on $1500, for 55 days, at 7½%.

9. Find the *exact* interest on $811.25, for 45 days, at 4½%.

10. Find the *exact* interest on $315, for 50 days, at 6%.

PERIODIC INTEREST.

435. Annual Interest is *simple interest* on the principal for each *year* period, and on each year's interest remaining unpaid.

436. Semi-Annual Interest is *simple interest* on the principal for each *half-year* period, and on each period's interest remaining unpaid.

437. Quarterly Interest is *simple interest* on the principal for each *quarter-year* period, and on each period's interest remaining unpaid.

As simple interest cannot be collected until the principal is due, simple and periodic interest are the same up to the end of the first interest period.

When the interest is not paid at the end of the periods, as agreed, much time will be saved in obtaining the amount due, by finding the interest on one over-due payment for the aggregate of the time for which they were all over-due; to this interest add the amount of the principal, at simple interest.

438. To find periodic interest, the principal, rate and time being given.

EXAMPLE.—What is the interest on $2500, from July 1, 1888, to Sept. 16, 1891, at 6% interest, due annually, and no payments made until final settlement?

OPERATION.

$$\begin{array}{r} 1891 — 9 — 16 \\ \underline{1888 — 7 — \ \ 1} \\ 3 — 2 — 15 = \text{time.} \end{array}$$

3 yrs. 2 mos. 15 ds. simple interest on principal = $481.25

$2500 × .06 = $150 = 1 yr. interest.

$$\text{Remaining unpaid for periods of} \left\{ \begin{array}{l} 2 \text{ yrs. } 2 \text{ mos. } 15 \text{ ds.} \\ 1 \text{ yr. } \ 2 \text{ mos. } 15 \text{ ds.} \\ \qquad\ \ \underline{2 \text{ mos. } 15 \text{ ds.}} \end{array} \right.$$

Interest of $150. for 3 yrs. 7 mos. 15 ds. = $ 32.63

Total interest due, = $513.88

EXPLANATION.—From July 1, 1888, to September 16, 1891, is 3 years, 2 months and 15 days. And since the first year's interest, which is $150, was not paid until 2 years, 2 months and 15 days after it was due, the second year's interest, $150, was not paid until 1 year, 2 months and 15 days after it was due, and the third year's interest, $150, was not paid until 2 months and 15 days after it was due, the aggregate of the time for which interest should be computed on 1 year's interest, $150, is 3 years, 7 months and 15 days, and its interest for that time is $32.63. The total interest is $481.25 + $32.63 = $513.88.

Rule.—*To the simple interest on the principal for the full time, add the interest on one period's interest for the aggregate of time for which the payments of interest were deferred.*

EXAMPLES.

439. *1.* What is the interest of $1260, payments due semi-annually from May 21, 1886, to Nov. 9, 1890, at 7%, no interest having been paid?

2. What is the interest of $3416.50, payments due quarterly from Jan. 15, 1884, to Sept. 6, 1891, at 5%, no interest having been paid?

3. Find the amount of interest due at the end of 4 yrs. 9 mos. on a note for $1155, at 6%, interest payable annually, but remaining unpaid.

4. On a note of $1750, dated Aug. 1, 1884, given with interest, at 10%, payable annually, the first three payments were made when due. How much remained unpaid, debt and interest, Jan. 1, 1891?

5. Find the amount due Oct. 11, 1891, on a debt of $11000 under date of July 5, 1888, bearing 4½% interest, payable quarterly, notes for the quarterly interest having been given and nothing paid until final settlement.

COMPOUND INTEREST.

440. Compound Interest is the interest on the principal and on the unpaid interest after it becomes due.

441. The **Unpaid Interest** may be added to the principal annually, semi-annually, quarterly, or for other agreed periods; and, when so added, interest is said to be compounded, *annually, quarterly*, etc., as the case may be.

Compound interest is not recoverable by law, but a creditor may receive it if tendered, without incurring the penalty of usury; a new obligation may be taken at the maturity of a compound interest claim, for the amount so shown to be due, and such new obligation will be valid and binding.

442. To find the compound interest when the principal, rate and time of compounding it are given.

EXAMPLE.—Find the compound interest of $750 for 3 yrs. 8 mos. 15 ds., at 6%, interest to be compounded annually.

OPERATION.

1st base = principal,	$750.00
1st year's interest =	45.00
2nd base = amount,	$795.00
2nd year's interest =	47.70
3rd base = amount,	$842.70
3rd year's interest =	50.56
4th base = amount,	$893.26
4th period's int., 8½ mos. =	37.96
Amount due on settlement,	$931.22
Less principal,	750.00
Compound interest =	$181.22

EXPLANATION.—Since the period for computing compound interest is for 1 year, add the first year's interest of $45 to the principal and the amount of $795 becomes the base upon which to calculate interest for the second year. The amount the second year, $842.70, becomes the base for the third year; the amount the third year, $893.26, becomes the base for the last period of 8 months and 15 days, the

amount of which is $931.22. As the amount $931.22, is the sum of the principal and interest, and as the principal is $750, the compound interest may now be found by subtracting the $750 from the amount; and $931.22 — $750 = $181.22, compound interest.

Rule.—I. *Find the amount of the principal for the first interest period; take this result as a principal for the next period, and so on through the whole time.*

II. *Subtract the principal from the last amount, and the remainder will be the compound interest.*

If the time contains fractional parts of a period, as months and days, find the amount due for the full periods, and to this add its interest for the months and days.

443. The labor of computing compound interest may be greatly shortened by the use of the following

COMPOUND INTEREST TABLE,

Showing the amount of $1 at compound interest at various rates per cent. for any number of years, from 1 year to 20 years, inclusive.

Yrs.	2½ per cent.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.0250 0000	1.0300 0000	1.0400 0000	1.0500 000	1.0600 000
2	1.0506 2500	1.0609 0000	1.0816 0000	1.1025 000	1.1236 000
3	1.0768 9062	1.0927 2700	1.1248 6400	1.1576 250	1.1910 160
4	1.1038 1289	1.1255 0881	1.1698 5856	1.2155 063	1.2624 770
5	1.1314 0821	1.1592 7407	1.2166 5290	1.2762 816	1.3382 256
6	1.1596 9342	1.1940 5230	1.2653 1902	1.3400 956	1.4185 191
7	1.1886 8575	1.2298 7387	1.3159 3178	1.4071 004	1.5036 303
8	1.2184 0290	1.2667 7008	1.3685 6905	1.4774 554	1.5938 481
9	1.2488 6297	1.3047 7318	1.4233 1181	1.5513 282	1.6894 790
10	1.2800 8454	1.3439 1638	1.4802 4428	1.6288 946	1.7908 477
11	1.3120 8666	1.3842 3387	1.5394 5406	1.7103 394	1.8982 986
12	1.3448 8882	1.4257 6089	1.6010 3222	1.7958 563	2.0121 965
13	1.3785 1104	1.4685 3371	1.6650 7351	1.8856 491	2.1329 283
14	1.4129 7382	1.5125 8972	1.7316 7645	1.9799 316	2.2609 040
15	1.4482 9817	1.5579 3742	1.8009 4351	2.0789 282	2.3965 582
16	1.4845 0562	1.6047 0644	1.8729 8125	2.1828 746	2.5403 517
17	1.5216 1826	1.6528 4763	1.9479 0050	2.2920 183	2.6927 728
18	1.5596 5872	1.7024 3306	2.0258 1652	2.4066 192	2.8543 392
19	1.5986 5019	1.7535 0605	2.1068 4918	2.5269 502	3.0255 995
20	1.6386 1644	1.8061 1123	2.1911 2314	2.6532 977	3.2071 355

For periods beyond the scope of the table: multiply together the amounts shown for periods the sum of which will equal the time required. For example, to find the compound amount of $1 for 40 years:

multiply the amount for 20 years by itself; to find the compound amount for 30 years: multiply the amount for 20 years by that for 10 years, etc.

To find the amount of any given principal, for any required number of years, multiply the given principal by the amount of $1.00 at the given rate as shown by the table.

If interest is to be compounded semi-annually, take one-half the rate for twice the time.

If interest is to be compounded quarterly, take one-fourth the rate for four times the time.

EXAMPLES.

444. *1.* What is the compound interest on $1200, for 4 years, at 6%, if the interest is compounded annually?

2. What is the compound interest on $600, for 3 years, at 5%, if the interest is compounded semi-annually?

3. What is the compound interest on $1640, for 1 yr. 6 mos., at 3%, if the interest is compounded quarterly?

4. Find the compound interest on $1000, for 4 yrs. 5 mos. 12 ds., at 8%, if the interest is compounded semi-annually.

5. What will be the amount due Feb. 11, 1892, on a debt of $900, bearing 10% interest, compounded quarterly, if the debt bears interest from July 1, 1888?

6. Oct. 1, 1888, I paid in full a note for $1350, dated March 15, 1883, and bearing 5% interest. If the interest was compounded semi-annually, what was the amount due at settlement?

MISCELLANEOUS EXAMPLES.

445. *1.* An account of $460 is past due 67 days. How much will the interest be at 6%?

2. $950 loaned at 7% for 1 yr., 8 mos., 18 days will amount to how much?

3. In what time will any sum of money double itself at 6%? at 5%? at 4%?

4. A note of $860 draws interest at 7½% from June 14th, 1890, to Aug. 5th, 1891. Find the amount due, calculating time by compound subtraction.

5. At what rate per cent. per annum must $1080 be loaned for 7 yrs., 3 mos., 27 ds., that it may amount to $1554.66?

6. In order to engage in business, I borrowed $3750 at 6%, and kept it until it amounted to $4560. How long did I keep the money?

7. A merchant sold a stock of glassware on one month's credit; the bill was not paid until 3 mos., 21 ds. after it became due, at which time the merchant received a draft for $4705.47 for the bill and interest thereon at the rate of 6%. Find the selling price of the goods.

8. December 11, 1887, a lumber dealer borrowed money and bought shingles at $4.50 per M; Sept. 17, 1888, he sold the shingles and paid his debt, and 8% interest, amounting to $3462.60. How many thousand shingles did he buy?

9. A merchant sold a bill of goods for $1800, payable without interest in three equal payments, in 3 months, 6 months, and 9 months, respectively. If money is worth 5% per annum, how much cash would be required for full settlement on the date of purchase?

10. I loaned a friend a sum of money for 9 months, at 6% per annum, and when the loan was due he paid $850.50 in cash, which was 75% of the amount due me; the remainder was paid 6 mos., 15 ds. later, with interest at the rate of 10%. Find the amount paid at final settlement.

11. Deland owns a summer resort valued at $45000, for which he receives, for a season of 5 months, $2400 rent per month. The year's expenses for taxes, repairs, and insurances, amount to $6375. If he sells this property and invests the proceeds in a manufacturing business paying quarterly $1937.50, how much will his rate of interest be increased by the change?

12. Having purchased 1150 barrels of pork, at $16 per barrel, on 4 months' credit, the dealer, 30 days later, sold it at $17.50 per barrel, receiving therefor a 6 months' note without interest. When the purchase money became due, he discounted the note on a basis of 7%, and paid his debt. How much was gained?

13. October 12, 1888, I purchased 2700 bushels of wheat, at $1.05 per bushel, and afterwards sold the wheat at a profit of 6%. On what date was the wheat sold, if my gain was equivalent to 10% interest on my investment?

14. A speculator borrowed $6250, at 7½% interest, and with the money bought a note, the face of which was $7500, maturing in nine months without interest, but which was not paid until two years from the date of its purchase. If the note drew

6% interest after maturity, did its purchaser gain or lose, and how much?

15. I am offered a house that rents for $27 per month, at such a price that, after paying $67.20 taxes, and other yearly expenses amounting to $24.85, my net income will be 8½% on my investment. What is the price asked for the house?

16. If money be worth 7%, compounded annually, which would be better, and how much, for a capitalist to loan $25000 for 11 years and 6 months, or to invest it in land that, at the end of the time named, will sell for $55000 above all expenses?

17. In a settlement with a merchant a man gave a note for $3450, payable in 1 year, 5 months and 21 days, with interest at 6% per annum. How much must be paid when the note becomes due?

18. On Dec. 6, 1891, a man received $238.37 in cash for the use of a certain sum of money which had been drawing 6% interest since July 15, 1890. Find the sum loaned.

19. A clothing dealer cleared during one year in business 15% on his investment. The following year he loaned his capital, less the gain, at 6%. At the end of the year his income was $2700 less than when in business. What was his capital?

20. A man invested $5500 at 6% simple interest until it amounted to $6061.00. How long was the amount invested and how long would it take to amount to $6622.00?

TRUE DISCOUNT.

446. True Discount is the difference between the face of a debt due at a future time and its *present worth.*

447. The **Present Worth** of a debt payable at a future time without interest is such a sum as, being put at simple interest at the legal rate, will amount to the given debt when it becomes due. Hence, the present worth of a debt is its value *now.*

To find *present worth,* apply the principles given in interest. The debt corresponds to the amount; the rate per cent. agreed upon to the rate; the time intervening before the maturity of the debt, to the time; and the present worth, which is the unknown term, to the principal.

When payments are to be made at different times, without interest, find the present worth of each payment separately, and take their sum.

With debts bearing interest, and discounted at the same or at a different rate of interest, the face of the debt plus its interest as due at maturity becomes the base.

448. To find the true discount and present worth of a debt.

EXAMPLE.—Find the present worth and true discount of a claim of $591.89 due in 4 months, 3 days without interest, money being worth 6% per annum.

OPERATION.

100% = present worth.
2.05% = true dist.
102.05% = debt = $591.89.
1% = $5.80.
100% = $580, present worth.
$591.89 − $580 = $11.89, true dist.

EXPLANATION.—Let 100% represent the present worth, 2.05%, the true dist. for 4 months, 3 days, and 102.05% will equal the debt of $591.89. As 102.05% = $591.89, 100% will equal $580, and $591.89 − $580 = $11.89, the true discount.

Proof: The interest on $580 for 4 months, 3 days = $2.90 × 4.1 = $11.89, and $580 + $11.89 = $591.89.

Rule.—*Divide the given amount of the debt by 100% plus the rate on 100% for the time, the quotient will represent the present worth; subtract the present worth from the debt, and the difference will be the true discount.*

EXAMPLES.

449. *1.* What is the present worth of $661.50, payable in 3 yrs. 9 mos., discounting at 6%?

2. Find the present worth and true discount of a debt of $138.85, due in 5 yrs. 6 mos. 18 ds., if money is worth 7% per annum?

3. Find the present worth of a debt of $1750, $1000 of which is due in 9 mos. and the remainder in 15 mos., money being worth 6% per annum?

4. Which is greater, and how much, the interest, or the true discount on $516, due in 1 yr., 8 mos., if money is worth 10% per annum?

5. Which is better, and how much, to buy flour at $6.75 per barrel on 6 months time, or to pay $6 cash, money being worth 6%?

6. When money is worth 5% per annum, which is preferable, to sell a house for $20,000 cash, or $21000 due in one year?

7. A farmer offered to sell a pair of horses for $420 cash, or for $475 due in 15 months without interest. If money is worth 8% per annum, how much would the buyer gain or lose by accepting the latter offer?

8. If money is worth 6%, what cash offer will be equivalent to an offer of $1546 for a bill of goods on 90 days' credit?

9. An agent paid $840 cash for a traction engine, and after holding it in stock for one year, sold it for $933.80, on eight months' credit. If money is worth 6%, what was his actual gain?

10. A stock of moquette carpeting, bought at $1.95 per yard, on 8 months' credit, was sold on the date of purchase for $1.80 per yard, cash. If money was worth 6% per annum, what per cent. of gain or loss did the seller realize or sustain?

11. A thresher is offered a new machine for $480 cash, $500 on 2 months' credit, or $525 on 1 year's credit. Which offer is the most advantageous for him, and how much better is it than the next best, with money worth 7½%?

12. After carrying a stock of silk for 4 months, I sold it at an advance of 30% on first cost, extending to the purchaser a credit of one year without interest. If money was worth 5% per annum, what was my per cent. of profit or loss?

BANKS AND BANK DISCOUNT.

450. A **Bank** is an instution, generally incorporated, for the receiving and loaning of money, for facilitating its transmission from one place to another, and, in case of banks of issue, for furnishing a paper circulation.

Money is transmitted from one place to another by means of checks, drafts and bills of exchange. Some banks perform only a part of the functions above mentioned.

451. Bank Discount is a deduction of the simple interest from the face or sum due upon negotiable paper at its maturity. The interest is calculated for the term of discount and at the legal or agreed rate for the cashing or buying of such paper before it becomes due.

452. The **Term of Discount** is the time the note has to run from the date of discount to maturity.

453. The **Proceeds of a Note** or other negotiable paper is the part left after the bank has deducted the bank discount from the maturing value.

454. Days of Grace are three days usually allowed by law for the payment of a note after the expiration of the time specified *in the note.* Banks charge interest for these three days.

In True Discount the present worth is the base; in Bank Discount the future worth is the base. As bank discount is simple interest taken in advance, aside from the days of grace, it differs from true discount in that it is interest on the true discount plus the true discount.

In general, notes discounted at banks do not bear interest. If the note be interest-bearing, the discount will be reckoned on and deducted from the amount due at maturity.

455. The **Maturity** of a note is the day on which it legally becomes due.

Notes containing an interest clause will bear interest from date to maturity, unless other time be specified.

Non-interest bearing notes become interest bearing if not paid at maturity.

456. To find the bank discount and proceeds of a note.

EXAMPLE.—Find the bank discount and proceeds of a note of $580 for 123 days at 6%.

OPERATION.

Principal = $580.
Rate = 6%.
Time = 4.1 mos.
Interest for 1 month = $2.90.
4.1 × $2.90 = $11.89, bank disct.
$580. − $11.89 = $568.11, proceeds.

EXPLANATION.— Since the interest for 1 month equals ½ of the principal in dollars considered as cents, and since 123 days equals 4.1 months, therefore, 4.1 × $2.90 = $11.89, the bank discount. As the bank discount is given, the proceeds are found by deducting the discount from the face of the note; and $580 — $11.89 = $568.11, proceeds.

Rule.—*Compute the interest for the given time at the given rate upon the value of the paper at maturity, the interest thus determined will be the bank discount; the bank discount subtracted from the maturity value of the note will give the proceeds.*

If the note be on interest, add to its face value the interest to maturity and discount the amount.

As a rule banks charge interest for the exact number of days, and in some cities both the day of date and day of maturity are included with the time. Usually bank notes are drawn for 30 days, 60 days, 90 days, etc., instead of 1 month, 2 months, 3 months, etc.

EXAMPLES.

457. *1.* Find the bank discount and proceeds of a note drawn for $1200, 90 days from date, and discounted at 6%.

2. Find the bank discount and proceeds of a note drawn for $1850, 30 days from date, and discounted at 6%.

3. A man sold a horse and received a 60-day note of $150 without interest, which note he sold at a bank at 10% discount, without recourse. If the horse cost him $120, how much did he make on the sale?

4. What is the proceeds of a note of $5000, drawn for 4 months, and discounted at 6%?

5. What is the bank discount on a note of $800, drawn for 6 months and discounted at 4½%?

6. Find the maturity, term of discount, and proceeds of a note of $400 bearing interest at 5% from Feb. 1, for a period of 6 months and discounted May 1 at 10%.

7. What is the difference between the true discount and the bank discount on a note of $530 on 1 year's time, not counting days of grace, money being worth 6% ?

8. Not counting days of grace, find the bank discount and proceeds of a note of $450 drawn for 8 years, 4 months, without interest, and discounted at 12%.

9. A dealer bought 100 brls. apples at $4 per brl. for cash. He sold 25 brls. for cash at $5 per brl., 25 brls. at $6 per brl. for a 30-day note without interest, and the remaining 50 brls. at $5.75 per brl. for a 60-day note drawing interest at 5%. He discounted the notes at 6% the same day as drawn, getting cash for the proceeds. How much did he make on the transaction?

10. The following note was discounted May 10th, 1891, at 6% :

$500. COLUMBUS, O., April 1, 1891.

Ninety days after date we promise to pay to the order of Hale & Goff, Five Hundred .... Dollars, with interest at 3 per cent.

M. J. BLOOD.

Find the discount and proceeds.

458. To find the face of a note.

EXAMPLE.—A man wishes to borrow $500 cash. For how much must he draw a bank note so that, when discounted at 6% on 90 days' time without interest, the proceeds will be the $500 wanted ?

OPERATION.

100% = face.
6% = rate of discount.
93 days = time.
$500 = proceeds.
$.0155 = disct. on $1 for 93 ds.
$1.00 — $.0155 = $.9845, proceeds $1.00.
$500 ÷ $.9845 = 507.87 times.
507.87 × $1.00 = $507.87, face.

EXPLANATION.—Since the discount on each $1.00 of the face for 93 days = $.0155 and the proceeds, ($1.00 — $.0155) = $.9845, the face of the note will be as many times $1.00 as $.9845, are contained times in $500, the proceeds of the face of the note. $500 ÷ $.9845 = 507.87 times; and 507.87 × $1.00 = $507.87, or face of the note.

Rule.—*Divide the proceeds of the note by the proceeds of $1.00 for the given time at the given rate per cent.; the result will equal the face of the note.*

EXAMPLES.

459. *1.* What must be the face of a 60-day note that, when discounted at 6%, the proceeds will be $497.50 ?

2. A banker's profit on discounting a certain note at 6% for 63 days was $15.75. What was the face of the note?

3. Find the face of a 4-months' 6% interest-bearing note that, when discounted at 6% for 123 days, the proceeds will be $499.79.

4. A dealer bought 50 brls. apples at $5.00 per barrel. For how much must he draw a 90-day note that, when discounted at 6%, he may pay the bill with the proceeds?

5. A 30-day 6% interest-bearing note had been on interest 3 days when the note was discounted for the remaining 30 days at 6%. If the bank discount was $40.22, what was the face of the note ?

460. To find the face of a note (short method).

Example.—A grocer wishes to receive $800 in cash as proceeds of his 90-day note. For what amount should he draw a 90-day note so that, when discounted at 6% for 93 days, the proceeds will be the amount desired ?

OPERATION.

Proceeds = $800.00
Int. on proceeds, 93 ds. = 12.40
Int. on $12.40, 93 ds. = .19
Face of note = $812.59

Explanation.—The difference between true discount and bank discount (except the 3 days of grace) is that bank discount is simple interest computed in advance on the face of the note, while true discount is *interest on the* present worth, for the time. The present worth of $812.59 is $800.19, and the true discount ($812.59 — $800.19) is $12.40. If now $812.59 be discounted for 93 days at 6%, the bank discount, $12.59, is the sum of the true discount, $12.40, and its interest for 93 days, or 19¢. Therefore, assume the proceeds of the note, $800, to be the present worth, then the interest on $800 for 93 days will represent the true discount, and the interest on the $12.40 for 93 days (19¢) will represent the difference between the true and bank discount as above explained. The sum of $800, $12.40 and $.19 = $812.59, the face of the note.

Rule.—*Add to the proceeds of the note the interest on the proceeds for the given time, and the interest on the interest for the given time. Their sum will represent the face of the note.*

EXAMPLES.

461. Find the face when the proceeds are as follows:

1. $600 for 33 days, at 6%.
2. $1500 for 63 days, at 6%.
3. $6000 for 33 days, at 6%.
4. $400 for 123 days, at 6%.
5. $500 for 93 days, at 6%.
6. $250 for 63 days, at 6%.
7. $450 for 33 days, at 7%.
8. $900 for 93 days, at 4½%.
9. $750 for 63 days, at 8%.
10. $1200 for 33 days, at 9%.

462. To find the proceeds of a note discounted when brokerage or collection is charged.

EXAMPLE.—Find the proceeds of a note of $8000 for 4 months, discounted through a broker at 6%, his brokerage being ¼%.

OPERATION.

Face = $8000.
Rate = 6%.
Brokerage = ¼%.
Time = 4.1 mos.
Interest 1 mo. = $40.
4.1 × $40 = $164.
Brokerage = $ 20.
Total, $184.
$8000 — $184 = $7816, proceeds.

EXPLANATION.—The interest for 1 month is $40, and for 4.1 months it is 4.1 times $40, or $164. The brokerage, ¼% of $8000 = $20, and the sum of $164 and $20, or $184, is the cost of discounting the note. The proceeds are $8000 — $184 = $7816.

Rule.—*From the face of the note subtract the discount for the time, plus the brokerage. The difference will be the proceeds.*

EXAMPLES.

463. Find the proceeds of the following notes, the brokerage being ⅛%.

1. $900, 123 days, dis. 6%.
2. $760, 93 days, dis. 4½%.
3. $5000, 63 days, dis. 5%.
4. $6000, 33 days, dis. 4%.
5. $9600, 63 days, dis. 10%.
6. $8000, 93 days, dis. 7½%.
7. $750, 63 days, dis. 6%.
8. $1750, 33 days, dis. 9%.
9. $8500, 93 days, dis. 6%.
10. $1600, 33 days, dis. 5%.

464. To find the legal date of maturity, the time being expressed in days.

EXAMPLE.—A note was made payable 90 days from January 10th, 1891. What was the legal date of maturity ?

OPERATION.

90 days = time.
3 days = grace.
93 days = time to run.
21 days remaining in Jan.
72 days remaining after Jan.
28 days in Feb.
44 days after Feb.
31 days in March.
13, April, date of maturity.

EXPLANATION.—Since 3 days of grace are allowed, the full time to run is 93 days from January 10th, 1891. The date of maturity is found by deducting from 93 days the remaining days in January, 21 days, and from the remainder after January the days in February, and from this remainder the days in March. The days remaining after March being 13, and less than the 30 days in April, the legal maturity of the note is therefore April 13th.

Rule.—*Add to the given time the 3 days of grace, the sum will be the days to run. From the days to run subtract the remaining days in that month in which the note is dated and from the remainder subtract the number of days in each succeeding month until a remainder is reached less than the number of days in the next month. The remainder thus found will represent the legal due date in that month.*

EXAMPLES.

465. Find the legal due dates on notes dated as follows:

1. Jan. 1, 1891, for 30 days.
2. May 10, 1891, for 60 days.
3. Apr. 30, 1891, for 90 days.
4. March 1, 1891, for 90 days.
5. April 10, 1891, for 30 days.
6. Feb. 14, 1891, for 90 days.
7. Jan. 13, 1891, for 120 days.
8. Jan. 25, 1891, for 90 days.
9. Apr. 19, 1891, for 60 days.
10. June 1, 1891, for 30 days.

466. To find the interest on daily balances.

EXAMPLE.—The Traders National Bank, of Georgetown, allows interest at 4% per annum to its depositors, on daily balances, payable quarterly. How much will the credit interest be on the following account current: Traders National Bank To Nelson Peckingham, Dr. *Jan. 1st, 1891,* deposited, $800; *Jan. 4,* checked out, $100; *8th,* deposited, $1000; *11th,* checked out, $500; *Feb. 10th,* deposited, $800; *20th,* checked out, $150; *March 5th,* deposited, $200; *30th,* checked out $50.

ACCOUNT CURRENT.

Date.	Deposits	Checks.	Balances.	Interest.	Ds.
1891.					
Jan. 1	800		800	.40	3
4		100	700	.47	4
8	1000		1700	.85	3
11		500	1200	6.00	30
Feb. 10	800		2000	3.33	10
20		150	1850	4.01	13
Mar. 5	200		2050	8.54	25
30		50	2000	.67	2
	2800	800	Bal. ℅ $2.000.00	3)24.27	
			16.18	8.09	

Balance due $2016.18

EXPLANATION.—By inspection it will be seen that $800 of the account was on interest for 3 days when on the 4th $100 was withdrawn by a check, leaving a balance of $700, which was on interest for 4 days when a deposit of $1000 was made. From Jan. 8th, when the deposit was made, the balance at that time, $1700, was on interest for 3 days, when on the 11th, $500 was checked out, leaving a balance of $1200, on interest for 30 days. The balances and the days to run for February and March are computed the same as in January. In the interest column are entered the items of interest on each balance for the days to run, computed for convenience on a 6% basis. The total interest for the month being $24.27, at 6%, deduct $\frac{1}{3}$ for 4%, and the remainder, $16.18, represents the sum due on balances at 4%. This $16.18, added to the balance of account, $2000, gives an amount due of $2016.18.

Rule.—*Compute the interest at 6% on each daily balance for the number of days it remains before another deposit is made or another amount checked out. The sum of the interest items will be the total amount due for interest at 6%.*

For any other rate than 6%, take such a part of the interest at 6% as the number of days is a part of 6 days.

MISCELLANEOUS EXAMPLES.

467. *1.* A note of $650 drawn for 60 days, without interest, was discounted for the full time to run at 6%. Find the bank discount and proceeds.

2. A note of $400, dated Mar. 13th, 1891, drawn for 60 days, without interest, was discounted at 6% on March 28th. Find date of maturity, bank discount and proceeds.

3. A note of $640, dated April 1st, 1891, drawn for 30 days, with interest at 5%, was discounted April 10th, at 6%. Find proceeds.

4. On Mar. 15th, 1891, John W. Allen borrowed of L. M.

Troup, for 4 months, $5000, on interest at 4½%, giving his note therefor. On April 16th, Troup discounted the note at a bank at 6%. Find legal date of maturity, bank discount and proceeds.

5. A grocer discounted his 30-day note at 6% discount and received $99.45 proceeds. Find the face of the note.

6. A New York broker receives from a correspondent a note dated June 15th, on 4 months time for $5000, on interest at 6%. If he discounts this at the bank on June 30th, for time to run at 4½%, how much should he remit after deducting his brokerage of ½%, and exchange on draft of ⅛%?

7. A note of $75000, dated Oct. 15, 1890, on 60 days' time, without interest, was sold on Oct. 18th by a broker at ⅜% commission and 4½% discount. He forwarded the proceeds in currency after prepaying charges, $12.50. Find the amount remitted.

8. For what shall a 90-day note be drawn, so that when discounted for 93 days, at 6%, the proceeds shall be $800?

9. Find the cash balance of the following account current with the First National Bank, Norton, O.: J. L. Brooks, Jan. 1, 1890, deposited, $500; Jan. 20, withdrew, $100; Mar. 16th, deposited, 500; Mar. 28, withdrew, $50. Interest 6% on balances, due April 1, 1890.

10. The Discount Register of the First National Bank, of Bolton, Ohio, shows the following entries at the close of business on April 25th, 1891, discount 6%.

No.	Date Note.	When Disc'td.	Maker.	When Due.	Face.		Disc't.		Proceeds.	
	1891.	1891.								
874	Mar. 1	Apr. 25	E. S. Snow	5/3	400		**	**	***	**
875	Apr. 25	Apr. 25	Thos. Brown	6/27	***		**	**	593	70
876	Jan. 1	Apr. 25	P. D. Redman	5/4	****		102	50	****	**
877	Apr. 25	Apr. 25	Moore & Co.	**	600		3	30	596	70
878	Apr. 25	Apr. 25	Cook & Co.	6/27	****		**	**	1484	25
879	Apr. 25	Apr. 25	Long & King	7/27	8000		**	**	7876	
880	Feb. 1	Apr. 25	Smith & Case	6/4	****		42	23	****	**
					****	**	***	**	****	**

In No. 874 find the discount and proceeds; in No. 875, face and discount; in No. 876, face and proceeds; in No. 877, when due and time; in No. 878, face and discount; in No. 879, discount; in No. 880, face and proceeds; also find total of each column for the day.

PARTIAL PAYMENTS.

468. A Partial Payment is a part payment of the face of a note, mortgage or other written obligation existing at the time such payment is made.

Part payments may apply to obligations, either before or after their maturity

Part payments are usually acknowledged, and should always be by indorsement on the back of the note or other obligation, but sometimes special receipts are given for the sums paid. Indorsements should give date and state amount paid; they are then equivalent to receipts.

Various rules are in use for finding the balance due on claims on which partial payments have been made; but only the United States Rule and the Merchants' Rule have more than *local* application.

469. The United States Rule is very generally used. It has the sanction of law, being the rule adopted by the Supreme Court of the United States, and has been adopted by most of the states.

470. The **Merchants' Rule** is used by most banks and business houses, where computations are on short time obligations, as such rule is regarded as the most convenient for business purposes.

The merchants' rule is varied in its use by different creditors, and hence is rather more an agreement, founded upon custom or otherwise, between debtor and creditor as to mode of settlement, than a strict rule of law.

471. The United States Rule for partial payments.

EXAMPLE.—A note dated Jan. 1, 1890, for $1500 bears interest for 1 year, 5 mos. and 3 days at 6% per annum. The following indorsements have been made upon it: March 16th, 1890, cash, $68.75; July 22, 1890, cash, $80.45; Dec. 31, 1890, cash, $25; March 1, 1891, cash, $500. What will be the amount due on June 14, 1891, calculating interest on a basis of 360 days to the year?

	Difference in Time.	Payments.		Interest.		Principal.
1890— 1— 1					Face note	$1500.00
1890— 3—16	= 2 mos. 15 ds.	$68.75	—	$18.75	= rem.	50.00
					rem.	$1450.00
1890— 7—22	= 4 mos. 6 ds.	$ 80.45	—	$30.45	= rem.	50.00
					rem.	$1400.00
1890—12—31	= 5 mos. 9 ds.	$ *25.00*		$*37.10*	} add	
1891— 3— 1	= 2 mos. 0 ds.	$*500.00*		$*14.00*		
		$525.00	—	$51.10	= rem.	473.90
					rem.	$ 926.10
1891— 6—14	= 3 mos. 13 ds.	$15.90, Interest				$ 15.90
	Amount due on settlement					$ 942.00

EXPLANATION.—The interest up to time of first payment, $18.75, is subtracted from the first payment, since, by legal decision "*the payment must be first applied to the discharge of the interest due;*" hence, $68.75 — $18.75 = $50.00, a remainder of payment, which remainder "*must* be applied to discharge of the *principal.*" Principal $1500 — remainder $50 = $1450, remainder of principal upon which interest is computed for the next period of 4 mos., 6 ds. The interest for the second period, $30.45, is subtracted from the payment of $80.45, leaving a remainder of $50, which is deducted from $1450, leaving $1400 remainder, upon which to compute interest for the next period of 5 mos., 9 ds. The interest for this period, $37.10, is *more* than the payment, $25, hence, "the balance of interest *must not be taken to increase the principal*" but interest continues on the *former* principal of $1400, for the next period of 2 mos., the interest of which is $14, when a payment of $500 is made. The payments of the 3d and 4th periods are now added to compare with the total interest for the *two periods* to see if the payments will now discharge the *interest* due. The aggregate of payments $525, will discharge the interest of $51.10, leaving a remainder of $473.90, to be deducted from the principal; $1400.00 — $473.90 = $926.10, now due. The interest for the last period of 3 mos., 13 ds., is $15.90. If now a settlement is made the amount due is $926.10 + $15.90 = $942.

Commencing with the date of the note arrange the dates in consecutive order, then by compound inverse subtraction determine the time between each two part payments and write to the right of each date the time and the payments made at each date.

Rule.—I. *Compute the interest for the time indicated to the time of the first payment, and write the result to the right of the first payment.*

II. *If the interest be less than the payment, subtract it from the payment, and subtract the remainder of the* *payment from the face of the note. The remainder of the principal will be the base upon which to compute interest for the second period. Compute the interest for the second period, and write the result at the right of the second payment, and proceed as before if the interest be less than the payment.*

III. *If at any time the interest be more than the payment, write the interest at the right of the payment, and compute the interest for the next period or periods until the aggregate sum of the payments exceeds the interest due; then subtract the remainder of payments from the remainder of principal and proceed as before.*

This rule is based upon the principle that neither payments nor interest shall bear interest, and when payments have been made the payment shall be applied in the first place to the discharge of the interest then due. The balance of the payments is applied to the discharge of the principal.

EXAMPLES.

472. Find the amount due at maturity on each of the following notes:

(*1.*)

$650.00. NEW ORLEANS, La., Jan. 1, 1891.

Six months after date I promise to pay to the order of S. C. Wilson, Six Hundred Fifty $\frac{00}{100}$ Dollars, with interest at 5%, value received. B. N. SHERMAN.

The above note was indorsed as follows: Feb. 10, 1891, cash, $53.52; March 12, cash, $11.00; Mar. 28, 1891, cash, $101.80; May 12, cash, $54.90; May 30, cash, $101.32.

(*2.*)

$1500.00. PITTSBURGH, Pa., Feb. 13, 1890.

One year from date, for value received, we, or either of us, promise to pay to the order of McMillan & Co., at Keystone National Bank, Pittsburgh, Pa., Fifteen Hundred Dollars, with interest at six per cent., without defalcation or discount.

JNO. C. BLOOD,
WM. D. BOTTOM.

Indorsed: May 13, 1890, cash, $500; June 19, 1890, cash, $200; Dec. 25, 1890, cash, $125.

(*3.*)

$975.00. San Francisco, Cal., Mar. 1, 1890.

One year from date, for value received, we promise to pay to the order of Post & Young, Nine Hundred Seventy-five Dollars, with interest at seven per cent. Payable at the Third National Bank.

Brooks & Porter.

Indorsed: May 7, 1890, cash, $87.51; June 22, 1890, cash, $5.00; Oct. 28, 1890, cash, $127.93; Jan. 4, 1891, cash, 58.80; Feb. 15, 1891, cash, $100.00.

(*4.*)

$350.00. Jacksonville, Fla., April 1, 1890.

Four months after date, for value received, I promise to pay to the order of Monroe & Smalley, Three Hundred Fifty $\frac{00}{100}$ *Dollars, with interest at eight per cent.*

Thomas Cook.

Indorsed: April 30, cash, $50.26; May 19, cash, $153.28; June 17, cash, $25.70; July 5, cash, $25.38; July 29, cash, $50.00.

(*5.*)

$2000.00. Tacoma, Wash., June 1, 1888.

Three years from date we promise to pay L. M. Goddard, or order, Two Thousand $\frac{00}{100}$ *Dollars, value received, with interest at ten (10) per cent.*

Poole & Dodge.

Indorsed: Sept. 1, 1888, cash, $20; Dec. 4, 1888, cash, $130.67; Jan. 15, 1889, cash, $93.45; March 18, 1889, cash, $933.25; May 27, 1891, cash, $500.

(*6.*)

$800.00. Cheyenne, Wyo., May 18, 1890.

One year from date, for value received, we promise to pay to the order of James M. Duncan, Eight Hundred $\frac{00}{100}$ *Dollars, with interest at twelve (12) per cent.*

M. C. Barnes,
Jas. Gleason.

Indorsed: Nov. 1, 1890, cash, $200; Jan. 4, 1891, cash, $25; Feb. 15, 1891, cash, $10; March 18, 1891, cash, $300.

473. The Merchants' Rule for partial payments.

EXAMPLE.—Find the balance due on Dec. 31, 1891, on a note dated Jan. 18, 1891, for $600 at 6% interest, and on which the following payments have been made: Feb. 15, cash, $25; March 18, cash, $75; April 15, cash, $50; May 20, cash, $50; June 21, cash, $75; July 14, cash, $125.

OPERATION.

Face of note, dated Jan. 18, 1891,			$600.00
Interest on same to settlement, 11 mos., 13 ds.,			34.30
Amount due on note,			$634.30
Payments.			
Paid February 15, cash,	$25.00		
Interest, 10 months, 16 days,	1.32	$26.32	
Paid March 18, cash,	75.00		
Interest, 9 months, 13 days,	3.54	78.54	
Paid April 15, cash,	50.00		
Interest, 8 months, 16 days,	2.13	52.13	
Paid May 20, cash,	50.00		
Interest, 7 months, 11 days,	1.84	51.84	
Paid June 21, cash,	75.00		
Interest, 6 months, 10 days,	2.38	77.38	
Paid July 14, cash,	125.00		
Interest, 5 months, 17 days,	3.48	128.48	
Total amount of payments,			414.69
Balance due on settlement,			$219.61

EXPLANATION.—Find the time by compound subtraction on the note from its date to settlement, which equals 11 months and 13 days, or $11.4\frac{1}{3}$ months. As the interest on $600 for 1 month = $3, for $11.4\frac{1}{3}$ months the interest = $11.4\frac{1}{3} \times \$3$, or $34.30, and this added to the face gives $634.30, amount of principal December 31, 1891. Finding the time from the date of each payment to date of settlement and calculating interest on each payment for the time indicated in the operation, the total amount of payments and interest thereon is $414.69, which, deducted from $634.30, leaves $219.61 due on settlement.

Rule.—*Find the amount of the principal to the time of settlement; also find the amount of each payment, from the time it was made to the time of settlement; subtract the amount of the payments from the amount of the principal debt; the remainder will be the balance due.*

Find time by compound subtraction. This rule is mainly used in cases of short time notes or business accounts, and by agreement.

EXAMPLES.

474. *1.* What balance is due June 1, 1890, on an account of $1450, bearing interest at 6% from Jan. 20th, 1890, on which payments have been made as follows: Feb. 1, $100; March 1, $150; April 1, $200; May 1, $500.

2. On March 15th, J. B. Brooks settled his account with Simons, Banning & Co. by giving a Due Bill for $853.50, with interest at 6%. How much was due on final settlement of the due bill, Oct. 1, the following indorsements having been made: April 1, $53.50; June 20, $100; Aug. 15, $500; Sept. 1, $175?

(*3.*)

$754.18. DENVER, Col., July 22, 1890.

On demand for value received, I promise to pay to the order of Jno. N. Bliss & Co., Seven Hundred Fifty-four and $\frac{18}{100}$ Dollars, with interest at 10%.

J. W. BLAIR.

Indorsed as follows: Aug. 1, $180; Sept. 10, $200; Dec. 20, $100. How much was due on settlement, Jan. 1, 1891?

4. Find the balance due at the maturity of the following note, payments having been made as follows: April 1, 1890, $500; Aug. 25, 1890, $1250; Nov. 3, 1890, $240; Dec. 30, 1890, $300; Feb. 1, 1891, $200.

$3000.00. ST. LOUIS, Mo., Dec. 3, 1889.

Eighteen months after date, I promise to pay to the order of Ezra R. Andrews, Three Thousand Dollars, with interest at 5 per cent.

Value received. GEO. J. BRAYER.

5. Find the balance due on the following note, payments having been made as follows: May 28, 1890, $255.50; June 13, 1890, $168.41; Aug. 31, 1890, $50; Sept. 28, 1890, $500; Nov. 1, 1890, $684.25.

$2150.00. DENVER, Colo., May 1, 1890.

Six months after date, I promise to pay to the order of Wm. H. Sanford, Two Thousand One Hundred Fifty Dollars, with interest at 8 per cent.

Value received. MARTIN F. RIGNEY.

EQUATION OF ACCOUNTS.

475. Equation of Accounts, or **Equation of Payments,** is the process of finding the date when one single payment may be made for two or more debts falling due on different dates; or the date when the *balance* of an account having both *debits* and *credits* may be paid without loss or gain of interest to either party.

This subject is also called Averaging Accounts or Averaging Payments.

476. The **Average Date of Payment,** or **Due Date,** is the date on which such payment or settlement may be equitably made. It is also called the Equated Date.

477. The **Focal Date** is any assumed date of settlement with which the dates of the several items in an account are compared for the purpose of finding the average time or due date.

478. The **Last Date** is usually taken for the focal date, and when so taken it represents the proper, instead of the assumed date of settlement.

479. The **Term of Credit** is the time to elapse before a debt becomes due; if given in days, it is counted on from the date of purchase or sale the *exact* number of days of the term; if given in months, it is counted on the number of months, regardless of the number of days thus included.

Book accounts bear legal interest after they become due, and notes, even if not containing an *interest clause*, bear interest after maturity.

480. General Rule.—*In all equations, extend time if credit or non-interest bearing time paper be involved; select the latest date as a focal date, find actual time in days, and compute interest at 6 per cent., on a 360-day basis.*

481. To find the equated time of an account when the items are on one side of the account and have no terms of credit.

EXAMPLE.—Render a statement of acct. for Robt. L. Brown, Ovid, to T. W. Blood, whose ledger account shows the following debits: 1890, Sept. 10th, mdse., $60; 26th, mdse., $100; Oct. 8th, mdse., $200; Nov. 1st, mdse., $120. Note at the bottom of the statement when due by equation, and state that a 60-day note may be given drawing interest at 6% from the equated date.

OPERATION.

Ovid, Nov. 1, 1890

T. W. Blood

To Robt. L. Brown, Dr.

Date.		Item.	Days.	Account.		Interest.	
1890.							
Sept.	10	Mdse.,	52	60			52
	26	Mdse.,	36	100			60
Oct.	8	Mdse.,	24	200			80
Nov.	1	Mdse.,	0	120			
				480		1	92

Due Oct. 8, 1890, payable by 60 da. note with use.

Explanation.—The acct. rendered shows a total debit of $480 on Nov. 1st, and all of the items except the last are past due. Since these sales on acct. were made without time, interest is due on them from date. On the first item the time from Sept. 10 to Nov. 1 is 52 days, and the interest on the bill of $60 for these 52 days is 52 cents. The days to run on the item of Sept. 26 are 36, the interest on the $100 for the 36 days is 60 cents. From Oct. 8 to Nov. 1 is 24 days, and the interest on the $200 for 24 days is 80 cents. On Nov. 1st Mr. Blood owes Mr. Brown the acct. of $480, and the interest thereon of $1.92. If now Mr. Blood wishes to settle the account of $480 without including the accumulated interest of $1.92, by giving an interest-bearing note, he must date the note as many days earlier than Nov. 1st as it will take days to produce $1.92 interest on $480. This may be determined as follows:

OPERATION.

$480 = principal.
$1.92 = interest.
6% = rate.
1 day = unit of measure.
Int. on $480 for 1 day = 8¢.
$1.92 ÷ $.08 = 24, or 24 ds.
24 days *before* Nov. 1 =
Oct. 8th, equated time.

Explanation. — The elements given are the *principal, interest* and *rate* to find the *time* in days. The *unit* of measure is 1 day. Since the interest on $480 for 1 day is $.08, it will take as many days to produce $1.92 as $.08 are contained *times* in $1.92; and $1.92 ÷ $.08 = 24 *times*; therefore 24 days are required to produce $1.92 interest on $480 as a base. If now the note be drawn 24 days before Nov. 1st and bearing interest at 6%, the note will be worth on Nov. 1st $480 + $1.92, or $481.92, the exact amount of the account and interest. Dating back 24 days from Nov. 1 gives Oct. 8th as the equated date.

Rule.—I. *To find the value of the account: Render a statement of account showing the total amount due up to and including the last item, the date of which use as the focal date.*

II. *Compute the interest on each item for the number of days from the date of the item to the focal date. The footing of the interest column will be the total amount due for interest, and the footing of the account and the interest together will represent the value of the account.*

III. *To find an equated date for the account without interest: Divide the total amount of accumulated interest by the interest on the account for 1 day, the quotient will represent the number of days required to produce the given interest on the total account due. Subtract the number of days required to produce the given interest from the focal date, and the date found will be the due date or date when settlement can be dated without loss to either party.*

Any *rate* may be taken as a standard and any *date* may be taken as the focal date, but the rate of 6% is the one most easily calculated, and the last date in the account is more in the line of settlement.

The *principle* involved in equations of accounts may be presented without casting interest; thus, in an account of three items, April 1, mdse., $100, April 11, mdse., $100, and April 21, mdse., $100, it may be readily seen that April 11th is the equated date, for while for April 1st the $100 is *past* due just 10 days, the item of April 21 of $100 would be paid just 10 days *before* being due, and hence the gain in the one case just equals the loss in the other.

EXAMPLES.

482. When are the following accounts due by equation as shown by Wood & Greenleaf's ledger?

(1)	T. W. Gray.	Dr.
1891.		
Jan. 1,	Mdse.,	$150
20,	Mdse.,	50
30,	Mdse.,	175
Mar. 1,	Mdse.,	350
31,	Mdse.,	50
		$775

(2)	Thos. W. Cooper.	Dr.
1891.		
June 1,	Mdse.,	$178
30,	Mdse.,	350
Aug. 15,	Mdse.,	322
		$850

(3)	M. B. Goswold.	Dr.
1891.		
Feb. 13,	Mdse.,	$648.10
28,	Mdse.,	115.50
Mar. 17,	Mdse.,	16.40
31,	Mdse.,	152.95
Apr. 30,	Mdse.,	110.05
		$1043.00

(4)	Brooks & Co.	Dr.
1891.		
Jan. 15,	Mdse.,	$2010.00
Mar. 18,	Mdse.,	150.14
June 19,	Mdse.,	1040.00
		$3200.14

(5)	D. M. Elwood.	Dr.
1891.		
Jan. 1,	Mdse.,	$1400
31,	Mdse.,	500
Feb. 10,	Mdse.,	600
20,	Mdse.,	400
		$2900

(6)	Douglass & Sharp.	Dr.
1891.		
Mar. 13,	Mdse.,	$142.80
30,	Mdse.,	175.20
Apr. 5,	Mdse.,	17.50
20,	Mdse.,	275.50
		$611.00

In finding the average time of credit in days, fractions of a day of one-half or greater are counted as a full day; fractions less than one-half are rejected. Carry out interest computations to four decimal places.

483. To find the equated time of an account, when the items are on one side of the account and have the same or different terms of credit.

EXAMPLE.—Render a statement of account on Sept. 1 for James Gleason & Co. to Babcock Bros., of the following debits: June 14, 1890, Mdse., 30 ds., $185; June 30th, Mdse., 60 ds., $175.14; July 5th, Mdse., net, $60; Aug. 1, Mdse., 30 ds., $314.26. Find the equated date for which a note may be drawn, bearing interest at 6%, in settlement of the above account.

OPERATION.

Monthly Statement.

Babcock Bros., Sept. 1, 1890.
To Gleason & Co., Dr.

Date.		Due.	Days.	Items.		Interest.	
1890.							
June	14	30 ds.	49	185		1	51
	30	60 ds.	3	175	14		09
July	5	net	58	60			58
Aug.	1	30 ds.	1	314	26		05
				734	40	2	23
Due	A	ug. 14th,					
by	n	ote 30 days.					

EXPLANATION. — The focal date is Sept. 1, when settlement was made. The first item, June 14th, $185, was on interest from June 14, to Sept. 1, *less 30 days*, as per term of credit, or 79 days—30 days=49 days. The time of the second item, June 30 to Sept. 1st, = 63 days — 60 days = 3 days for which to compute interest. The third item being a net cash bill is on interest for the full time of 58 days. The last item is dated Aug. 31, and the time is 31 days — 30 days or 1 day, for which to charge interest. The face of the account is $734.40, and the interest on the account is $2.23. The value of the account on Sept. 1 is $734 40 + $2.23 = $736.63. Now, to find the equated date, it will be necessary first to divide the given interest by the interest on the face amount of the account for 1 day; the interest on \$734.40 for 1 day = \$.1224. As the interest for 1 day is \$.1224, and the total interest is \$2.23, it will take as many days to produce \$2.23 as \$.1224 is contained times in it; and \$2.23 ÷ \$.1224 = 18 times, or 18 days; and 18 days earlier than Sept. 1 = Aug. 14, the equated date.

It will be seen by inspection that the item of June 14, \$185, is due 30 days later, or July 14; the second item Aug. 29; the third, having no term of credit, is due at date of sale; the last, Aug. 1, \$314.26, is due Aug. 31. This date, Aug. 31, could be made the focal date, but where settlement is made at a later date than the latest due date, the date of settlement is often used as a focal date.

In business, odd days, odd cents, and even odd dollars, are often rejected in the interest calculations in equating the time. But for class work, *exact* money, *exact* time, and interest computed to *four* decimal places, should be required.

Rule.—*Use for the focal date the latest due date, or the date of settlement if later. Determine the time on each item in the account for which to charge interest by counting the number of days from the time each item falls due to the focal date, and proceed to compute interest and find the equated date as directed in Art. 481.*

EXAMPLES.

484. Find the equated date of the following accounts of Jerome, Simmons & Co.

(1) Genung Bros. & Co. Dr.

1891.		
Jan. 1,	Mdse., 30 ds.	\$500
30,	Mdse., 60 ds.	500
July 29,	Mdse., net	500
Aug. 28,	Mdse., 30 ds.	500
		\$2000

(2) Hanover & Gibbs. Dr.

1891.		
Jan. 10,	Mdse., 60 ds.	\$500
Feb. 9,	Mdse., net	500
Mar. 18,	Mdse., 30 ds.	450
Apr. 17,	Mdse., 30 ds.	450
		\$1900

(3) Warren & Camp. Dr.

1891.		
May 10,	Mdse., net	\$150.00
31,	Mdse., 30 ds.	225.00
June 17,	Mdse., 120 ds.	175.00
30,	Mdse., net	480.00
		\$1030.00

(4) F. W. Blair. Dr.

1891.		
June 10,	Mdse., net	\$510.00
June 30,	Mdse., 30 ds.	375.50
Aug. 1,	Mdse., 4 mos.	250.00
Sept. 1,	Mdse., 60 ds.	450.00
		\$1585.50

(5) Byron, Judd & Co. Dr.

1891.			
Jan.	1,	Mdse., 30 ds.	$150.75
	20,	Mdse., 60 ds.	200.25
Mar.	1,	Mdse., 90 ds.	300.00
	20,	Mdse., net	125.00
			$776.00

(6) W. C. Duvan & Co. Dr.

1891.			
Mar.	1,	Mdse., 30 ds.	$575.80
Apr.	14,	Mdse., 60 ds.	250.17
May	17,	Mdse., 30 ds.	100.14
June	19,	Mdse., 90 ds.	75.25
			$1001.36

485. To find the equated date when an account has debits and credits, with or without terms of credit.

EXAMPLE.—James W. Goodspeed owes Martin & Co. on acct. as follows: Jan. 25th, Mdse., net, $650; Feb. 21st, 1891, Mdse., 30 ds. $400; Mar. 5th, Mdse., 30 ds., $125; Mar. 5th, Mdse., 60 ds., $225. Paid on same, Jan. 23d, note 30 ds., without interest, $300; Mar. 1st, note, 60 days on interest, $500.

Render statement on May 4, showing equated date.

MONTHLY STATEMENT RENDERED.

PIERPORT, O., May 4, 1891.

Jas. W. Goodspeed,
In acct. with Martin & Co.

Date.		Due.	Explanations.	Days.	Items.		Interest.	
1891.			Dr.					
Jan.	25	net	Mdse.,	99	650		10	725
Feb.	21	30 ds.	Mdse.,	42	400		2	80
Mar.	5	30 ds.	Mdse.,	30	125			625
	5	60 ds.	Mdse.,	0	225			
			Cr.		1400		14	15
Jan.	23	30 ds.	Note, no int.	68	300		3	40
Mar.	1	60 ds.	Note, on int.	64	500		5	33
					800		8	73
Debit balance due March 11, 1891.					600		5	42
To be settled by 90 day note.								

EXPLANATION.—The last debit item is due on May 4th, 1891. The total debit is $1400, and the debit interest is $14.15. The total credit is $800, and interest on payments, $8.73. The balance due on the account therefore is $600, and excess of debit interest over the credit interest is $5.42. The value of the account, therefore, on May 4, is $605.42. The interest on $600 for 1 day at 6% is 10¢; and $5.42 ÷ 10¢ = 54 times, or 54 days. The equated date therefore is 54 days earlier than the focal date, or March 11th, 1891, the time when settlement can be dated without loss to either party.

Rule.—*Assume for focal date the latest due date. Compute the interest on the debit items and on the credit items of the account. If the excess of interest and balance of account be upon the same side date back; if on opposite sides date forward.*

EXAMPLES.

486. *1.* Find the equated date of payment.

Dr. THOMAS W. BLANCHARD. *Cr.*

1891.				1891.			
Jan.	1	Mdse.,	600	Jan.	10	Cash,	500
	10	Mdse.,	400		30	Cash,	400
	30	Mdse.,	500	Feb.	15	Cash,	200
Mar.	1	Mdse.,	1000				

2. When is the following account due by equation?

Dr. GEO. H. BAKER. *Cr.*

1891.				1891.			
Feb.	15	Mdse., net	750	Mar.	1	Cash,	700
	28	Mdse., 30 ds.	818	Mar.	10	Note, 30 ds.,	
Mar.	10	Mdse., 60 ds.	215			on interest,	500
June	1	Mdse., net	217	Mar.	31	Cash,	155

3. Find the equated date of payment ?

Dr. P. L. GOODRICH. *Cr.*

1891.					1891.			
Jan.	8	Mdse., net	1550		Jan.	30	Cash,	200
Feb.	10	Mdse., 4 mos.	875		Feb.	28	Note, 30 ds.,	
Mar.	17	Mdse., net	685	50			no interest,	500
Apr.	12	Mdse., 30 ds.	175	50	Mar.	18	Cash,	1000

In case a negotiable paper is given, its maturity is determined in the usual way, by adding to its express time three days of grace. If the paper bear interest, its *value* is equivalent to its *face* as *cash* at its date; while if the paper be non-interest bearing, its *value* is equivalent to cash at its maturity.

4. Find the balance due and when due by equation.

Dr. Elmer W. Williams. *Cr.*

1891.					1891.				
Jan.	15	Mdse., 30 ds.	1800		Feb.	17	Cash,	2000	
Feb.	13	Mdse., 60 ds.	1750		Mar.	25	Cash,	1000	
Mar.	20	Mdse., 6 mos.	1185		June	10	Note, 4 mos.,		
Apr.	17	Mdse., net	675				on interest,	2000	
May	25	Mdse., 90 ds.	43						
June	30	Mdse., 60 ds.	217						
July	5	Mdse., 30 ds.	1878						
Aug.	1	Mdse., net	2592						

5. Find the balance of the following account, the interest balance and when due by equation, money being worth worth 6%.

Dr. Edwin H. Sprague. *Cr.*

1891.					1891.				
May	11	Mdse., 2 mo.,	108	31	June	1	Cash,	124	27
July	1	Mdse., 30 ds.	225		Oct.	31	4 mo. note (on		
Aug.	31	Mdse., net	280	80			interest),	167	91
Oct.	1	Mdse., net	137	50	Dec.	1	Cash,	305	05

6. Find, 1st, when the following account is due by equation; 2nd, the cash balance due Jan. 1, 1892, if money be worth 6%; 3rd, the value of the account at focal date.

Dr. Benj. S. Bascomb. *Cr.*

1890.					1891.				
Dec.	15	Mdse.,	200		Jan.	2	Cash,	300	
Dec.	28	Mdse., 2 mos.	300		Mar.	1	60 ds. note (no		
1891.							interest),	150	
Jan.	14	Mdse., 30 ds.	300						

7. Find the balance due and when due by equation.

Dr. Theodore E. Pond. *Cr.*

1890.					1890.				
May	14	Mdse., 1 mo.	300		May	31	2-mo. note (no		
June	3	Mdse., 60 ds.	200				interest),	240	
July	31	Mdse., 2 mos.	400		July	15	30-da. note (on		
							interest),	150	
					1891.				
					Jan.	1	Cash,	100	

8. Find balance due and when due by equation.

Dr. L. M. Ensign & Co. *Cr.*

1890.					1890.				
Nov.	1	Mdse., 2 mos.	150		Nov.	1	Cash,	200	
Dec.	3	Mdse., 3 mos.	150		Dec.	1	3-mo. accept.		
Dec.	14	Mdse., 60 ds.	150				(no interest),	200	
1891.					1891.				
Jan.	15	Mdse.,	300		Feb.	15	Cash,	200	

9. Find balance due, value of the account at focal date and when due by equation, money being worth 6%.

Dr. Paulding & Marion. *Cr.*

1890.					1890.				
Oct.	15	Mdse., 30 ds.	278	50	Nov.	20	Cash,	210	
Oct.	31	Mdse., 4 mos.	147	50	Dec.	31	2 mo. accept.		
1891.							(no interest),	175	
Jan.	5	Mdse.,	160	25	1891.				
Feb.	14	Mdse., 60 ds.	311	50	Mar.	1	60-da. note		
							(on interest),	220	50

10. When are the net proceeds of the following account sales due by equation?

Dunkirk, Pa., Nov. 30, 1890.

Account Sales Flour, For account of Graves & Co.,
By Jas. C. Burton. Burlington, Iowa.

1891.		**Sales.**					
Sept.	20	95 brls. Flour, cash,	$5.60				
Oct.	1	200 brls. Flour, 1 mo.,	$5.75				
	18	65 brls. Flour, 60 days,	$5.75				
Nov.	3	110 brls. Flour, 30 days,	$5.80				
	25	130 brls. Flour, cash,	$5.75				
		Charges.					
Sept.	24	Freight,		62	50		
	26	Cartage,		30			
Oct.	28	Cash, advanced,		2000			
Nov.	5	Cooperage,		5			
	25	Commission,		137	78		
		Net proceeds,					
		Due by equation,................189......					

In rendering *Account Sales*, the expenses (freight, storage, commission, etc.) charged constitute the *Debits* of the account. and the gross sales constitute the *Credits*. Equate such accounts in the usual manner.

CUSTOM HOUSE BUSINESS.

487. Duties or **Customs** are taxes levied by the Government on imported goods for revenue purposes and for protection of home industries.

Duties are of two kinds, *ad valorem* and *specific.*

488. An **Ad Valorem Duty** is a certain per cent. charged or levied on the cost of the goods as shown by the invoice from the country exporting them. Ad valorem means *according to value.*

489. A Specific Duty is a tax assessed upon the number, weight, or measure of goods, regardless of their value; as a fixed sum per bale, ton, barrel, etc.

490. Tonnage is a tax levied upon a vessel independent of its cargo, for the privilege of coming into port.

Tare, Leakage, Breakage, etc., are allowances made before estimating specific duties.

491. Tare is an allowance made for the weight of the box, bag, crate or other covering of the goods.

492. Leakage is an allowance made for the waste of liquids imported in barrels or casks. It is determined by gauging.

493. Breakage is an allowance made for the loss of liquids imported in bottles.

494. Weights. The Gross Weight is the weight before any allowances are made. Net Weight is the weight after all allowances have been made.

The ton used at the custom house is the long ton of 2240 pounds.

Duties are not computed on fractions of a dollar; if the cents are less than 50 they are rejected; if 50 or more they are counted as another dollar.

Warehousing is depositing imported goods in a government or bonded warehouse.

A *bonded warehouse* is used for storing goods on which the duties have not been paid.

Goods may be withdrawn from a bonded warehouse for export, without the payment of the duties. If goods on which the duty has been paid are exported, the amount of duty so paid is refunded; the sum so refunded is called a *drawback.*

495. To find the specific or ad volorem duty, or both, the invoice and rate being given.

EXAMPLE.—Find the specific duty on 40 casks of alcohol of 60 gallons each at 15 cents per gallon, leakage 5%; and the ad valorem duty of 35% on 90 boxes of brass rivets of 25 pounds each, invoiced at 12¢ per pound; tare, 6 pounds per box.

OPERATION.

1st invoice:—
Gross quantity = 2400 gals.
Leakage, 5% = 120 gals.
Net quantity = 2280 gals.
Specific duty =
2280 × 15¢ = $342.00
2nd invoice:—
Gross weight = 2250 #
Tare, 6 # per box = 540 #
Net weight = 1710 #
Value = 1710 × 12¢ = $205.20
Ad. val. duty =
205 × 35¢ = $ 71.75
Total duty = $413.75

EXPLANATION.—Since specific duty is computed on the net quantity, deduct the leakage, 120 gals., from the gross quantity, 2400 gals., and the difference, 2280 gals. will be the net quantity. As the duty on 1 gal. is 15¢, on 2280 gals., it will be $342, the duty on the first invoice. Since ad valorem duty is computed on the net value, find the net weight by deducting the tare, 540 lbs., from the gross weight, 2250 lbs., which will leave 1710 lbs. net. The net value of 1710 lbs. at 12¢ per pound = $205.20; and 35% of $205 (no fraction) equals $71.75, the duty on the second invoice. The total duty equals $342 + $71.75, or $413.75.

Rule.—I. *Deduct all allowances from the gross quantity and multiply the net quantity by the duty per single article of the kind or class considered; the product will be the specific duty.*

II. *Multiply the net value, after all deductions have been made, by the rate per cent. of duty assessed; the product will be the ad valorem duty.*

EXAMPLES.

496. *1.* What is the ad valorem duty, at 50%, upon a consignment of 650 dozen kid gloves, invoiced at 90 francs per doz.?

2. An importation of English crockery was invoiced at £896, 5s. 6d. Find the ad valorem duty, at 40%.

3. If an importation of goods is invoiced at 17450 francs, what will be the ad valorem duty, at 35% ?

4. If the ad valorem duty on sperm oil is 20%, what will it amount to in an importation of 600 barrels, of 42 gallons each, invoiced at 45¢ per gallon, 3½% being allowed for leakage ?

5. What is the duty on 1000 yds. of brussels carpet, 27 in. wide, invoiced at 6s. 9d. per yd.; duty 44¢ per square yard specific, and 35% ad valorem ?

6. If the duty on flannel is 29¢ per pound specific, and 35% ad valorem, how much must be paid on an invoice of 2150 yds., weighing 420 lbs., and valued in Canada, whence it was imported, at 75¢ per yard ?

7. Find the duty on 3 dozen clocks, invoiced at $21.50 each, and 6 dozen watches, invoiced at $35 each, if the ad valorem duty was 35% on the clocks, and 25% on the watches.

8. How much duty must be paid on an importation of 27640 lb. of wool, invoiced at £1497 10s. 4d., if the rate of duty is 10¢ per pound specific, and 11% ad valorem ?

9. An agent imported from Canada 7240 bushels of barley, and 17½ tons of hay, invoiced at $9.50 per ton. What amount of duties had he to pay, at 10¢ per bushel on the barley and 20% on the hay ?

10. A merchant imported 300 pieces of three-ply carpet, each piece containing 75 sq. yds., invoiced at 3s. 6d. per square yard, upon which he paid a duty of 17¢ per square yard specific, and 35% ad valorem. What was the total amount of duty paid?

11. An invoice of woolen cloth, imported from England, was valued at £956 6s. If its weight was 684 lbs., how much was the duty, at 50¢ per pound specific, and 35% ad valorem ?

12. Find the duty on 50 cases of tobacco, each weighing 60 lbs., worth $1.00 per lb., and 50000 Havana cigars weighing 550 lbs., invoiced at $75 per M, the duty being 50¢ per pound specific on the tobacco and $2.50 per pound specific on the cigars, and 25% ad valorem on both.

TAXES.

497. Taxes are sums of money levied on persons, property, or products for any public purpose.

498. Poll Taxes are taxes levied at a certain amount for each person, or head, of legal voters not exempt by law.

499. Property Tax is a tax assessed or levied upon property at a given rate per cent. of the *assessed* valuation. Property is of two kinds, Personal and Real.

500. Personal Property is *movable* property; as merchandise, ships, money, stocks, mortgages, goods, etc.

501. Real Property or **Real Estate** consists of *immovable* property; as houses and lands.

Assessors are public officers who appraise the value of property to be taxed. Collectors are officers who collect taxes.

Taxes are generally assessed and made payable in money; but "road taxes" may be made payable in "day's" work.

502. To find a property tax, the assessed valuation and rate per cent. of tax being given.

EXAMPLE.—The rate of taxation in the City of Grand Island, Neb., was 1¾% for the year 1890. What was the amount of tax that a person paid who owned personal property assessed at $7500, and real estate assessed at $10900 ?

OPERATION.

Personal property = $ 7500
Real estate = 10900
100% = valuation = $18400
1¾% of valuation = .01¾ × $18400 = $322.

EXPLANATION.—The valuation = 100%, and the rate = 1¾% of valuation, therefore, 1¾% of valuation, or $7500 + $10900, = .01¾ × $18400 = $322.

Rule.—*Multiply the total assessed valuation by the rate per cent. of taxation.*

EXAMPLES.

503. *1.* A person having lands assessed at $12250 and personal property at $6000, pays a tax of 1½%. Find his total tax.

2. H. W. Bowen has real property assessed at \$750 and personal property at \$22500. He pays a tax on all of $1\frac{5}{8}\%$. What is his total tax?

3. R. Bradshaw, of Rutland, has a farm assessed at \$4000; stocks, bonds, and chattels at \$8500. Find the amount of his taxes, the rate being $\frac{5}{8}\%$ on total assessed valuation.

4. George H. Beemis has real estate worth \$60000, assessed at \$38000 and taxed at $1\frac{3}{8}\%$. How much must he pay annually?

In some of the states the custom is not to tax personal property except in cases where parties are engaged in business.

Real estate is seldom or never assessed at its full or cash standard value, and all assessments are reviewed by a board usually selected from the county commissioners. In city valuation in New York the city assessors review their own record when appealed to by taxpayers.

5. Gordon & Bronson's store and land is assessed at \$17500, stock of groceries at \$1850, and horses, wagons, etc., at \$1275. The rate of taxation is $\frac{7}{8}\%$. Find their total tax.

503. To find a general tax, the valuation and amount of tax being given.

EXAMPLE.—The county commissioners of Saline County, Kansas, have levied a total tax upon all the real and personal property of the county, for state, school and county purposes, to the amount of \$8550. The total assessed value of property in the county is: personal property, \$251550; real property, \$508450. What will be the rate of taxation, and how much will W. L. Banker pay, who owns personal property to the assessed value of \$8600?

OPERATION.

Personal property = \$251550
Real property = 508450
Total valuation = \$760000 = 100%.
Total tax to be raised = \$8550.
1% of \$760000 = \$7600.
\$8550 ÷ \$7600 = $1\frac{1}{8}$, or $1\frac{1}{8}\%$.

W. L. Banker's tax.

100% = \$8600.
$1\frac{1}{8}\%$ = tax rate.
$1\frac{1}{8}\%$ of \$8600 = $.01\frac{1}{8} \times \$8600 = \$96.75$.

EXPLANATION.—The total assessed value of the property of the county represents the base, or 100%. The amount of tax to be raised represents the percentage. The rate per cent. of taxation represents the rate. 1% of the total valuation, \$760000 = .01×\$760000=\$7600. As the amount of tax levied is \$8550, the rate will be as many per cent. as 1% of the base or $7600 is contained *times* in $8550; and $8550 ÷ $7600 = 1⅛ times; and 1⅛ × 1% = 1⅛%, or rate of taxation. The tax of W. L. Banker equals 1⅛% of $8600 = .01⅛ × $8600 = $96.75.

Rule.—I. *Represent the total assessed valuation of real and personal property as the base or 100%, and the amount of tax to be raised on property, as the percentage. Divide the amount of tax by 1% of the assessed valuation and the quotient will represent times 1% of rate.*

II. *Multiply each person's total assessed valuation by the rate per cent. of tax, the product will be tax to be paid.*

In case there is a poll tax, deduct the poll tax from the total tax, the remainder will be the net total tax to be raised on property under the rule.

EXAMPLES.

504. *1.* A tax of $125000 is levied on a city, the assessed valuation of which is $15000000. What is the rate of taxation, and what amount of tax will a person have to pay whose property is assessed at $7500 ?

2. If a tax of $120 is assessed on a mill valued at $24000, what is the assessed valuation of a residence that is taxed $17.75 at the same rate ?

3. The per cent. of tax assessed for state purposes is ⅞%, for county ½%, and for city 1⅛%. What will be the amount of my tax, on property assessed at $21500 ?

4. The tax assessed upon a town is $20914.80; the town contains 2580 polls, taxed $.62½ each, and has an assessed real estate valuation of $4062000, and of personal property to the amount of $227400. Find the rate of taxation, and C's tax, who pays for 1 poll, and whose property is assessed at $15000.

5. For the year 1888 the rates of taxation in the State of New York were as follows: Schools, 1.085 mills; general purposes, 1.475 mills; new capitol, .6 of a mill; other purposes, .34 of a mill. What was the total rate of taxation, and how much was raised by Livingston county, the assessed valuation of which, as fixed by the State board of equalization, was $25395180 ? How much did said county raise for school purposes ?

INSURANCE.

505. Insurance is indemnity against loss or damage. It is of two kinds: *Personal Insurance* and *Property Insurance.*

506. Personal Insurance is the insurance of persons. It is of three kinds: *Life, Accident* and *Health* insurance.

507. Property Insurance includes :

508. Fire Insurance, or indemnity for loss of or damage to property by fire.

509. Marine Insurance, or indemnity for loss of or damage to a ship or its cargo, by any specific casualty at sea or on inland waters.

510. Live Stock Insurance, or indemnity for loss of or damage to horses, cattle, etc., from lightning or other casualty.

511. The **Insured Party** is usually the owner of the property insured, but may be any person having a financial *insurable* interest in the property.

512. The **Insuring Parties** are called *Insurers* or *Underwriters,* and are usually incorporated companies.

513. Insurance Companies are distinguished by the way in which they are organized; as *Stock Insurance Companies, Mutual Insurance Companies.*

514. A **Stock Insurance Company** is one whose capital has been contributed and is owned by stockholders, who share the profits and are liable for the losses.

515. A **Mutual Insurance Company** is one in which the profits and losses are shared by the insured parties.

Some companies combine the features of both stock and mutual companies, and are called *Mixed Companies.*

In mixed companies, all profits above a limited dividend to the stockholders are divided among the policy-holders.

516. Transit Insurance refers to risks taken on goods being transported from place to place, either by rail or water or both.

517. The **Policy** is the contract between the insurance company and the person insured or the person whose property is insured, and contains a description of the person or property insured, the amount of the insurance, and the conditions under which the risk is taken.

518. The **Premium** is the consideration in the contract, or the sum paid for insurance.

519. The **Term of Insurance** is the period of time for which the risk is taken.

Insurance companies usually reserve the privilege of rebuilding, replacing, or repairing damaged property.

Insurance policies ordinarily state that the loss, if becoming a charge upon the company, will be paid *30 days* or *60 days* after due notice and proof of loss. If not then paid, the amount of the claim becomes interest bearing.

520. **Salvage** is an allowance made to those rendering voluntary aid in saving vessels or cargoes from marine casualties.

521. The computations in Property Insurance are performed the same as in Percentage, and the terms compare as follows:

The Amount Insured = the Base; the Rate Per Cent. of Premium = the Rate; the Premium = the Percentage.

522. To find the cost of insurance.

EXAMPLE.—A grocer insures his stock of goods for $2500 at 1¼% premium. Find cost of insurance, if $1.00 is charged for the policy.

OPERATION.

$2500 = amount insured.
1¼% = rate.
.01¼ × $2500 = $31.25 premium.
$1.00 policy.
Total cost, $32.25

EXPLANATION.—The amount insured is the base, therefore, 1¼% of $2500 = .01¼ × $2500 = $31.25, premium, and $31.25 plus $1, the cost of the policy, = $32.25, the total cost of insurance.

Rule.—*Multiply the amount of insurance by the rate per cent. of premium, and add extra charges, if any; the sum will be the total cost of insurance.*

EXAMPLES.

523. *1.* A stock of goods is valued at $1400. A policy is taken out for ¾ of its value in the Hartford Ins. Co. at 1⅛% premium. Find the cost of insurance.

2. How much insurance may be obtained at 1⅛% for $72.45 premium?

3. A business man insured ⅘ of the inventory value of his house at 1½%, the premium being $60. What was the value of his house?

4. A speculator had in storage 200 brls. of apples, the value of which he insured at 2%. He paid $16 for premium. How much per barrel was the value of the apples?

5. John L. Moodey insures his house for $3500 at 1½%; horses, cows and other live stock for $800 at ¾%; household goods, $2000, at 1½%; stock of groceries for $2800 at 1⅛%. Find total cost of insurance.

6. The Hartford Insurance Co. insured the Blanchard block, valued at $50000 for $40000, at 1¼% premium. It reinsured $25000 in the Ætna at ⅝%. How much was the net gain to the Hartford?

7. An insurance agent received 20% of all premiums on new business for the year 1890. His income was $1140 and the rate of insurance was 1½%. What was the amount of property insured?

8. Henry Livingston owns ⅖ of a steamboat which is insured at ⅔ of its value at 1½%. The total cost to Livingston for premium was $480. What was the value of the steamer?

9. The contents of a factory was insured for a certain sum at 1½%. Later the goods were damaged by fire and losses paid by the company to the amount of $18750, which was ⅔ of the amount insured. If the amount insured was ¾ of the value inventoried, what was the total value of the goods?

10. Banks & Sherwin insured their stock of goods for $50000, placed as follows: Ætna, $12000, at 1⅛%; Hartford, $10000, at 1⅛%; Phœnix, $8000, at 1⅛%; Manhattan, $10000, at 1⅛%; Provident, $10000, at 1⅛%. Shortly afterwards the store and contents were destroyed by fire. Before a board of insurance adjusters it was determined that the firm had suffered a loss of $40500. Find the proportionate amount to be paid by each company.

STOCKS AND BONDS.

524. Stocks is a general term applied to both the shares of a corporation and to the bonds issued by government.

525. A Stock Certificate is a written or printed instrument of a Joint Stock Company or Corporation, signed by the officers of the company, certifying that the holder of the certificate is the owner of a certain number of shares of its capital stock.

A *share* simply represents a certain component part of the capital stock, which is usually divided into shares of $25, $50, $100, $1000. The Stock Certificate represents the number of shares specified therein.

The Capital Stock of a company is the sum of all the shares issued, at their *par* value.

The Par Value of stock is the sum for which stock is issued. The Market Value is the sum for which stock can be sold.

Stocks are at *par* when they sell for the value written on their face.

Stocks are *below* par when worth less than their face value, and *above* par when worth more than their face value.

526. The **Preferred Stock** of a corporation is given to secure some obligation of the company and upon it dividends are declared in preference to the common stock.

Preferred Stock is usually issued to take up certain floating indebtness of a corporation, and agreed dividends are declared at certain intervals, the dividends often being charged up to expense account without regard to the net earnings. Such stock is usually issued upon the reorganization of railroads and consolidated joint stock companies.

527. Watered Stock is stock issued to stockholders in addition to what they own, and for which no extra payment is required.

When the charter of a corporation or the general law governing corporations, forbids them to declare dividends in excess of a certain *per cent.*, or when the stockholders wish to keep the public in ignorance of their prosperity, watered stock is issued and dividends are declared upon the inflated basis. Thus, if upon a capital stock of 2000 shares, 8% is the limit, and the loss and gain account shows at least 12% profit, 1000 shares of watered stock would be issued, then 8% dividend would be declared upon the 3000 shares, and the 12% profit on the 2000 shares be thus absorbed.

528. A **Bond** is a written or printed obligation of government, Joint Stock Company or Corporation. It is conditioned to pay a certain sum of money at a specified time and at an agreed or fixed rate of interest, payable at regular intervals.

Bonds of Corporations are usually secured by First Mortgage on their real estate. *Municipal Bonds* are issued by a vote of the people or their representatives, and for the payment of which a Sinking Fund is accumulated by a yearly rate per cent. levied on all the real property within the limits of the municipality.

529. A **Coupon Bond** is one with certificates attached showing amount, date of interest and when due. When paid the coupons are detached and cancelled as vouchers.

530. A **Registered** bond is one which is payable to the owner as registered in the books of the corporation or government issuing it. Registered bonds can be transferred only by assignment and registry on the books of the corporation.

531. **Government Bonds** are bonds issued by the general government. Their names are usually derived from the interest they bear and the time when due; as the 4-Twenties, 4½-'91's, etc.

532. A **Corporation** is a fictitious person. It consists of several natural persons, who in the name of the corporation are authorized by law to transact business.

The instrument which defines the rights and duties of a corporation is called a *Charter*. It is issued by government, under seal.

533. **Stock Quotations** are the public prices or rate per share that stock sells for.

534. A **Dividend** is a certain profit divided among stockholders.

Dividends and assessments are declared at a certain per cent. on the capital stock. Dividends are declared yearly, semi-annually and quarterly.

535. An **Assessment** is a sum levied upon the stockholders of a corporation to make up its losses, etc.

536. **Premium** is the per cent. profit of stock over 100%, or its par value. Discount is the per cent. loss of stock below 100%, or par value.

537. To find the dividend on stocks, the capital stock and rate per cent. of dividend being given.

EXAMPLE. — The Hamilton Machine Co., of Bridgeport, Conn., has a net profit of $17812.50 for the year, to be adjusted. As its capital stock is $25000, divided into 2500 shares of $100 each, and the directors have declared a dividend of 5½%, how much will the dividend be, and how much will the sinking fund be after the dividend is transferred? also, how much of the dividend will be due A. B. Jones, who owns 15 shares?

OPERATION.

Capital = $250000.
Rate of dividend = 5½%.
Net earnings = $17812.50.
Dividend = .05½ × $250000 = $13750.
Sinking fund =
$17812.50 — $13750 = $4062.50.

A B. Jones' dividend.

Value 15 shares = $1500.
5½% of $1500 = $82.50.

EXPLANATION.— Since the dividend is a certain per cent. of the capital stock, 5½% dividend represents 5½% of $250000 = $13750. As the net earnings are $17812.50, and the dividends are $13750, there will remain for the sinking fund, the difference between the two amounts; and $17812.50 — $13750 = $4062.50. Since A. B. Jones has 15 shares, the par value of which is $1500, his dividend will be 5½% of $1500 = $82.50.

Rule.—I. *To find the total dividend: Multiply the par value of the capital stock by the rate per cent. of dividend.*

II. *To find the dividend due a stockholder: Multiply the par value of one share by the number of shares owned, and multiply this product by the rate per cent. of dividend.*

EXAMPLES.

538. *1.* A bank with a capital of $200000 declares a dividend of 4½%. Find the amount of dividend, and B's dividend who owns 12 shares, of the par value of $100 each.

2. An Ohio railroad company having a capital stock of $3500000, divided into 35000 shares, declares a semi-annual dividend of 2¾%. How much is the total dividend, and how much is due A, who holds a certificate for 16 shares?

3. The Orange Gas Co., of New Jersey, declares a dividend of 7½% on its capital stock of $300000. Find the total dividend and the amount due Martin Bishop, who owns 25 shares of the par value of $50 each.

4. The Jenkins Machine Co., of Richmond, Va., declared a quarterly dividend of 1¾% on its capital stock of $150000. Find the total dividend due stockholders, and C's portion, who owns 50 shares of the par value of $100 each.

5. A manufacturing concern has a capital stock of $200000. Its net earnings for the year are $28512.75. Pursuant to its charter 3% of the net earnings is set aside as a Sinking Fund to cover losses, 8% is declared for dividends, and the remainder is transferred to an account called Undivided Profits, from which to declare subsequent dividends or to meet current expenses. Find the amount of dividend account, sinking fund account, and undivided profits account.

539. To find the rate per cent. of dividend, the capital stock and net earnings being given.

EXAMPLE.—An insurance company having a capital stock of $500000, has net earnings of $45813.50. After setting aside 1½% for a sinking fund, how great an even per cent. dividend may be declared out of the remainder ?

OPERATION.

Capital stock = 500000.00
Net earnings = 45813.50
Sinking fund, 1½% = 7500.00
Remainder, $38313.50
1% of capital = $5000
$38313.50 ÷ $5000 = 7 times,
Plus $3313.50 undivided profits.
Rate dividend = 7%

PROOF.

Sinking fund, 1½% = $ 7500.00
Dividend, 7% = 35000.00
Undivided profits = 3313.50
Net earnings = $45813.50

EXPLANATION.—Since the rate per cent. of dividend is a certain number of hundredths of the capital stock, the capital stock is the base. As 1½% is to be set aside from the net earnings for a sinking fund, which equals $7500, then $45813.50 — $7500 = $38313.50, the remainder from which to declare dividends. As 1% of the capital stock = $5000, there can be as great an even per cent. dividend declared as $5000 is contained an even number of times in $38313.50; and $38313.50 ÷ $5000 = 7 times, with a remainder of $3313.50; therefore the rate per cent. of dividend is 7% and undivided profits = $3313.50.

Rule.—*From the net earnings deduct the amount to be set aside for the sinking fund, if any, and divide the remainder by 1% of the capital stock; the quotient will represent the rate per cent. of dividend.*

EXAMPLES.

540. *1.* A railroad company having a capital of $400000 has net earnings of $33500 to divide among its shareholders. What is the greatest even per cent. of dividend that may be declared?

2. A gas company having 2000 shares, at the par value of $50 each, has net earnings of $18500. What even rate per cent. of dividend may be declared?

3. If the capital is $1500000, and the net earnings are $52500, what rate per cent. of dividend may be declared?

4. On Jan. 1st, 1890, a manufacturing company issued 2000 shares of its capital stock at the par value of $100 each. At the end of the year the net earnings were $16850. What even rate per cent. of dividend was declared, and how much remained for undivided profits?

5. The Dan'l Dunning Glove Co.'s loss and gain accounts show the following for the year: Gross earnings, $29518.75; expenses, $9618.75. Its capital stock being $200000, what even per cent. dividend may be declared, and what will be the amount of undivided profits, if 1% is first set aside as a sinking fund?

541. To find the par value, the premium or discount being given.

EXAMPLE.—Sold Metropolitan Gas Co. stock for $18000 at a gain of 12½%. Find the par value of the stock.

OPERATION.

100 % = par value.
12½% = premium.
112½% = sale, $18000.
1 % = $160 unit measure.
100 % = $16000, par value.

EXPLANATION.—The par value is 100% and the gain, 12½%, is the premium, therefore, 112½% = sale of $18000. As 112½% = $18000, the unit of measure, or 1% = $18000 ÷ 112½ = $160; and 100% or par value = 100 × $160 = $16000.

Rule.—*Divide the sale by 100% plus the rate of premium, or minus the rate of discount; the quotient will represent 1% of the par value, which multiplied by 100, will represent 100%, or the par value.*

When the net gain or the net loss, and the per cent. of gain or loss are given, divide the gain or loss by the rate and multiply the quotient by 100.

When dividends or assessments and the amount of the sale are given proceed as under the rule.

EXAMPLES.

542. Find the par value.

1. $16640 sale, 4% premium.
2. $24000 sale, 4% discount.
3. $17510 sale, 3% premium.
4. $82616 sale, 8% loss.
5. $17940 sale, 8% loss.
6. $750 assessment, 5% rate.
7. $1450 dividend, 2% rate.
8. $640 discount, 4% rate.
9. $845 loss, 2½ rate.
10. $648 gain, 3% rate.

543. To find the market value, the premium or discount being given.

Example.—Find the market value of 150 shares Illinois Central Railroad stock, par value $100 each, quoted at 93½%.

OPERATION.

Par value = 150 × $100 = $15000.
100% = par value.
93½% = market value.
.93½ × $15000 = $14025 market value.

Explanation.—The par value of 1 share is $100. The par value of 150 shares equals 150 × $100 = $15000. 100% = the par value, and the market value = 93½% of the par value, therefore .93½ × $15000 = $14025, market value.

Rule. *Multiply the par value by 100% plus the rate per cent. of premium or minus the rate per cent. of discount. The product will be the market value.*

EXAMPLES.

544. Find the market value of the following stock of the par value of $100 per share.

1. 50 shares quoted at 92¾.
2. 75 shares quoted at 31½.
3. 80 shares quoted at 75⅜.
4. 100 shares quoted at 43½.
5. 200 shares quoted at 72.
6. 58 shares quoted at 62½.
7. 150 shares quoted at 85.
8. 300 shares quoted at 73.
9. 500 shares quoted at 154⅛.
10. 450 shares quoted at 112½.

545. To find the rate per cent. of investment, the cost and dividend being given.

Example.—What per cent. profit is made on stock costing 80 and paying a dividend of 10%?

OPERATION.

80% = cost.
10% = dividend.
1% = unit of measure.
1% of 80% = .8%.
10% ÷ .8 = 12½%.

Explanation.—To find the rate of investment refers to the cost, therefore, 1% of cost = 1% of 80%, or .8%. As 1% of cost equals .8% profit on the investment, 10% profit will equal as many per cent. as .8% is contained times in 10%; and 10% ÷ .8 = 12½, or 12½%. Proof: 12½% of 80% = .12½ × 80% = 10%.

Rule.—*Divide the rate of income by 1% of the cost or market value. The quotient will represent the rate of investment.*

EXAMPLES.

546. Find the rate of investment.

Rate of Income.	Cost.	Rate of Income.	Quotations.
1. 12½%.	75.	*6.* 16⅔%.	83⅓.
2. 10%.	90.	*7.* 10%.	80.
3. 33%.	110.	*8.* 6%.	95.
4. 20%.	120.	*9.* 5%.	96.
5. 40%.	130.	*10.* 15%.	120.

MISCELLANEOUS EXAMPLES.

547. *1.* What per cent. profit is made on the investment if stock paying 6% dividend is bought at 120?

2. A manufacturing company having a capital of $100000 has net earnings of $6500. What rate per cent. dividend may be declared?

3. A natural gas company declares a semi-annual dividend of 7½% on its capital of $300000. How much is due C, who owns 68 shares of the par value of $50 each?

4. How much money will be required to purchase 150 shares M., K. & T. railroad stock, par value $100 each, quoted at 76¼%, brokerage ⅛%?

5. I paid $4440 for Kansas City & Pacific Railroad stock at 74. How many shares at $50 each did I buy?

6. H. M. Brockway bought through a broker, at ⅜% brokerage, the following shares of railroad stock of the par value of $100 each: Mobile & Ohio, at 65⅝, 75 shares; Milwaukee & St. Paul, at 123, 100 shares; Louisville & Nashville, at 113¼, 80 shares; N. Y. Central, at 123¼, 20 shares. Find the total cost.

7. A broker bought for a customer, at ⅛% brokerage, 800 shares New Jersey Central Railroad stock, par value $100, at a total cost to his customer of $100100. Find the market quotation and brokerage.

8. A business man owns 18 shares of gas stock of the par value of $100 per share, upon which he was paid $99 as a dividend. If the capital stock was $150000, what was the total dividend paid to stockholders?

EXCHANGE.

548. **Exchange** is a system by which debts are paid in distant places without the transmission of money. This is done by the means of written orders called Bank Drafts, Bills of Exchange, Commercial Paper, Express Orders, Telegraphic Money Orders, Postal Notes and Post Office Money Orders.

549. **Bills of Exchange** are classed as Domestic or Inland, and Foreign.

550. **Domestic Bills of Exchange** are those payable at some place in the same country in which they are drawn.

Domestic Bills are called Drafts whether drawn at sight or on time.

551. **Foreign Bills of Exchange** are those payable at some place in another country.

Foreign Bills are usually drawn in sets, called First and Second or First, Second and Third, according as they are drawn in sets of two or three.

552. A **Place of Exchange** is some great money center.

The principal centers of exchange in the United States are New York, Boston, Philadelphia, Chicago, St. Louis, Baltimore, Cincinnati and San Francisco. Those of Europe are London, Paris, Antwerp, Geneva, Amsterdam, Hamburg, Frankford, Bremen, Berlin and Vienna.

553. A **Bank Draft** or Bill of Exchange is an order written by one bank directing another bank to pay a specified sum of money to a third party or to his order. Following is a form of

BANK DRAFT.

GERMAN-AMERICAN BANK, No. 1849.
MANCHESTER, June 7, 1891.

Pay to the order of
……………… *E. W. Hooker* ……………… $155.86
One Hundred Fifty-five ……………… $\frac{86}{100}$ *Dollars.*
To Ninth National Bank,
New York.

THOS. J. BANNISTER,
Cashier.

Nearly all banks keep money deposited with some one bank, called a correspondent, at their nearest commercial center, against which they draw drafts to sell to their customers for remittance to creditors, these drafts pass as cash in the section of country tributary to the commercial center where the bank is located upon which the draft is drawn.

554. **Commercial Paper** consists of Sight and Time Drafts drawn by one person or firm, called the drawer, directing a second person or firm, called the drawee, to pay to a third person or firm, called the payee, or to his order, a specified sum of money. Following is a common form of

COMMERCIAL PAPER.

$418.50. UTICA, N. Y., June, 1, 1891.
At one day's sight
Pay to the order of F. M. Barlow & Son,
Four Hundred Eighteen..........and $\frac{50}{100}$ *Dollars,*
and charge to the account of
B. W. SLOCUM & Co.
To Buchanan Bros.,
Newark, N. J.

Bank drafts are drawn against cash balances on deposit, while commercial drafts are drawn to collect a debt due. One is called cash when drawn, the other is called cash when collected.

555. An **Express Money Order** is an order drawn by the agent of an express company at the office where the order is bought, directing another agent of the company, at some place designated, to pay to the person named therein a certain sum of money.

Express orders are transferable by indorsement the same as checks.

EXPRESS MONEY ORDER RATES.

Payable in U. S. or Canada.		Payable in Europe.	
For not over		For not over	
$ 5.00	5¢.	$10.00	10¢.
$10.00	8¢.	$20.00	18¢.
$20.00	10¢.	$30.00	25¢.
$30.00	12¢.	$40.00	35¢.
$40.00	15¢.	$50.00	45¢.
$50.00	20¢.	Exceeding $50 at same rates.	

Value of £1 sterling, $4.90; 1 franc, $.20; 1 mark, $.24.

The above and following tables of rates are given to aid the student in working the examples relating to money orders in this subject. These rates, while current at the time of the writing of this book, are subject to change, and hence should not be regarded as authentic by the business man, until verified.

556. Telegraphic Money Orders represent a system of exchange by which the remitter sends a message to the payee, directing him to call at a certain telegraph office for the sum named therein. The telegraph agent at the sending office instructs the agent at the receiving office, to pay to the person named in the message, the sum specified, upon his personal application and proper identification.

Express companies also contract to transmit orders for funds by telegraph, but instead of requiring the payee to call for the funds in person, they deliver them to him at his residence or place of business.

TELEGRAPHIC TRANSFER RATES.

For not more than $50, 50¢; $50 to $100, 1%; $100 to $200, $1.25; over $200 to $300, $1.50; over $300 to $400, $1.75; over $400 to $500, $2.00; over $500, special rates.

The above rates are in addition to cost of telegraphic service, which is based upon distance and the number of words contained in the message.

557. United States Post Office Money Orders are drawn by the postmaster, or his clerk, at one office, directing the postmaster of another office to pay to the person named in his private letter of advice, the sum specified in the order.

Money orders can be transferred but once and then by indorsement only. Not more than three orders of not to exceed $100 each can be issued to the same person payable at the same office on the same day.

U. S. MONEY ORDER RATES.

Domestic.		International.	
For sums not exceeding		For sums not exceeding	
$ 5	5¢.	$10	10¢.
$10	8¢.	$20	20¢.
$15	10¢.	$30	30¢.
$30	15¢.	$40	40¢.
$40	20¢.	$50	50¢.
$50	25¢.	$60	60¢.
$60	30¢.	$70	70¢.
$70	35¢.	$80	80¢.
$80	40¢.	$90	90¢.
$100	45¢.	$100	$1.00.

Value of £1, sterling, $4.87.

558. Postal Notes are issued for sums not exceeding $4.99. Being payable to bearer they are unsafe to transmit through the mails.

The cost of a postal note is 3 cents, in addition to the sum specified on its face.

559. The **Par of Exchange** is the established value or equivalent of the standard unit of money of one country, expressed in the standard unit of money of another country. It is of two kinds, *intrinsic* and *commercial.*

The intrinsic par value refers to bullion value. Thus, the pound sterling of Great Britian contains 113 grains of pure gold, and the dollar of the United States contains 23.22 grains pure gold. Since 113 grains are 4.8665 times 23.22 grains, the dollar is worth 100 cents, and the pound is worth 486.65 cents.

The commercial par value refers to the value of the coin or currency of one country compared with that of another country, as determined by its market value, or by the requirements of trade or commerce. Thus, while the present bullion value of £1 is $4.8665, the value of £1 in express exchange is $4.90, in United States Money Orders, $4.87.

DOMESTIC EXCHANGE.

560. To find the cost of domestic exchange.

EXAMPLE.—A hardware merchant of Dayton, Ohio, owes P. Morton & Co., of Bridgeport, Conn., an account of $750. He buys of his banker a draft on the Commercial Bank of New York, at a cost of 10¢ per hundred. How much did the draft cost?

OPERATION.

Draft, face = $750.00
Exchange = 7½ × 10¢ = .75
Cost of draft, = $750.75

EXPLANATION.—The cost of the draft will be its face, $750, plus the cost of exchange, 75¢ or $750.75.

Credit will be given by Morton & Co., however, for only $750, for should they deposit this draft with their banker, or transfer it to any one on account, the draft would be worth only its face, $750.

Rule.—*Add to the face of the draft, the charge for drawing same; the sum will represent the cost of the draft.*

Formerly domestic exchange was at a premium or discount in the city where purchased, according as the balance of trade between that city and the one on which the draft was drawn, was in favor of or against the former city. If the drawer city owed the drawee city, exchange on the latter would be at a premium in the former. If the balance of trade was in favor of the drawer city, the conditions of exchange would be reversed in the two places.

For several years past, however, domestic exchange has been practically at par throughout this country. Bankers usually make a charge, called "exchange," for the trouble of keeping funds at commercial centers and drawing drafts against these funds, and also one, called "collection," for collecting drafts deposited with them payable at banks outside of the section in which the banks receiving them are located, or where they have no regular correspondents. Some bank drafts, as those drawn on New York, are current at par almost everywhere in this country.

Some banks make no charge for domestic exchange to regular depositors.

Commercial paper is generally bought and sold at a discount, the rate depending somewhat upon the cost of transmitting the money from the place where the paper is purchased to the place on which it is drawn, and the condition of the money market in the former place.

Instead of making remittances by exchange, merchants often send their personal checks in payment of bills. These, having to be returned for collection, command a small charge, as 15¢ or 20¢ on each check, and not a *per cent.* on the amount.

561. To find the proceeds of commercial paper drawn at sight.

EXAMPLE.—Barnum & Co., of St. Paul, drew a sight draft for $1400 on Martin & Cole, 415 High St., Boston, on account of an invoice of hides shipped them, valued at $3000, as per B. of L. attached to draft. They sold the draft at a bank at ½% discount. What were the proceeds?

OPERATION.

Face of draft = $1400
Dis. = ½% of $1400 = 7
Proceeds = $1393

EXPLANATION.—As the draft is at sight, no time is considered, and as the B. of L., valued at $3000 is attached to the draft, that, together with the indorsement of Barnum & Co., is a sufficient guarantee to the bank for the paper. The discount is ½% of $1400 or $7, which subtracted from the face of the draft leaves $1393, the proceeds.

Rule.—*Multiply the face of the draft by the rate per cent. of discount, the product will be the discount. Subtract the discount from the face of the draft, the result will be the proceeds.*

Or, multiply the face of the draft by 100%, minus the rate of discount.

562. To find the proceeds of commercial paper drawn on time.

EXAMPLE.—A commission merchant of Charleston, S. C., bought a 90-day commercial draft for $800 on a Boston firm at ⅛% discount. If money be worth 6%, what did the draft cost him?

OPERATION.

Bank dis. on \$1.00 for 93 ds. = \$.0155
Commercial dis. on \$1.00 = \$.005
Total dis. on \$1.00 = \$.0205
Proceeds of \$1.00 = \$1.00 − \$.0205 = \$.9795
\$800 draft costs 800 × \$.9795 = \$783.60

EXPLANATION.—As the bank discount on \$1.00 for 93 days is \$.0155 and the commercial discount on \$1.00 at ½% is \$.005, the total discount on \$1.00 is \$.0205, and the proceeds are \$1.00 — \$.0205=\$.9795. Since each \$1.00 of the draft cost \$.9795, the face of the draft will cost 800 × \$.9795 = \$783.60.

Rule.—*Find the cost of \$1.00 by deducting the bank discount for the time and the commercial discount at the given rate. Multiply the cost of \$1.00 by the face of the draft, the product will represent the cost of the draft.*

563. To find the cost of exchange by Express money orders, U. S. money orders and Telegraphic money orders.

EXAMPLE.—W. J. Boone & Co., of Bandon, Oregon, have bills to pay as follows: T. W. Brooks, Dayton, O., \$650; E. L. Greyson & Sons, Rosedale, Oregon, \$46.53; Barnes & Snyder, Bolton, Mo., \$48.50; and their traveling salesman, W. H. Post, is wanting \$100 for expenses at Denver, Colorado. They pay the amounts by remitting as follows: T. W. Brooks and E. L. Greyson & Sons, express money orders; Barnes & Snyder, P. O. money order, and W. H. Post by telegraphic transfer in a 10-word message, the message costing 50¢. Find the total cost of the remittances.

OPERATION.

Express orders:
Dayton remittance, \$650.00
Exchange, 2.60 = \$652.60
Rosedale remittance, \$ 46.53
Exchange, .20 = \$ 46.73
U. S. money order:
Bolton remittance, \$ 48.50
Commission, 25 = \$ 48.75
Telegraphic transfer:
Agent's remittance, \$100.00
commission, 1%, 1.00
cost of message, .50 = \$101.50
Total, \$849.58

EXPLANATION.—By the table of rates under Art. 555, the cost of \$650 in express order is \$2.60 (20¢ on each \$50) and the total cost of the remittance to Dayton is therefore \$652.60, and to Rosedale, \$46.73. The commission on U. S. money orders being 25¢ for sums over \$40 and less than \$50, the order on Bolton costs \$48.75. The rate for sending money by telegraph being 1% on sums of not more than \$100, and the cost of the service being 50¢, the total cost of the remittance to the agent at Denver is $101.50. The total cost of exchange for the bills paid is $849.58.

Rule.—I. *To find the cost of a remittance by Express order or P. O. money order: Add to the sum remitted the exchange as determined by the table of rates.*

II. *To find the cost of telegraphic transfer of money: Add to the sum transferred the commission, as determined by the table, and the cost of sending the message.*

FOREIGN EXCHANGE.

564. To find the cost of a foreign bill of exchange, the face of the bill and the rate of exchange being given.

EXAMPLE.—Find the cost of a draft on London for £380, 10s. 6d. sterling, exchange being quoted at $4.86⅝.

OPERATION.

Face draft = £380.525.
Monetary unit = £1.
Market quotation of £1 = $4.86⅝.
380.525 × $4.86⅝ = $1851.729.
Cost of draft = $1851.73.

EXPLANATION.—Changing 10s. 6d. to the decimal of a pound = £$\frac{126}{240}$ = £.525, and the face of the draft in sterling exchange is worth £380.525. The value of £1 is $4.86⅝, therefore, £380.525 is worth 380.525 × $4.86⅝ = $1851.73.

Rule.—*Change the given denominations to units and the decimal of a unit of the money of account of the country upon which the exchange is drawn. Multiply the value of 1 monetary unit, or market quotation, by the number of monetary units, the product will represent the cost in United States money.*

The following quotations of the foreign moneys of account are used as the basis for comparisons in the United States: Great Britain, £1 = $4.8665; France, 1 Franc = $.193; German Empire, 1 Mark = $.238; Spain, 1 Peseta = $.193; Italy, 1 Lira = $.193; Mexico, 1 Dollar = $.79; Brazil, 1 Milreis = $.546; Cuba, 1 Peso = $.932; Hayti, 1 Gourde = $.965.

Computations in the following examples will depend upon the quotations given in each.

EXAMPLES.

565. *1.* Find the cost of a bill of exchange on Paris for 15000 francs, at $.196.

2. An importer owes a bill in London of £400, 8s. 10d. How much will it cost him to settle the bill in London exchange, quoted at $4.86⅝?

3. A grocer at Kingsboro, Pa., had a debt of $38.17, due in New York, which he paid by U. S. money order. Find the cost of the order.

4. Find the cost of a bank draft for $2580 on Third National Bank of New York, at the rate of 10 cents exchange for each $100.

5. Gates & Son, of Memphis, drew a sight draft on Perrin & Boon, of Portland, Me., for $81750.85, which they sold at the Cotton Exchange Bank at $\frac{5}{8}$% discount. How much were the proceeds?

6. Jno. W. Williams, of Middlesex, Mass., remitted Janis Bros. & Co., of Milwaukee, $1750 by draft on New York, exchange 15 cents per each $100; Martin & Co., of Allentown, Pa., by American Express money order, $89.75; and Theodore Emens, $28.50, by P. O. money order. Find the total cost of exchange.

7. L. C. Thompson, of St. Louis, bought of Glenn & Carson a 90-day draft for $3800 on Leroy & Co., of Boston, at $\frac{3}{4}$% discount and less 6% interest for the time to run. He sent the draft to a Boston broker who sold it at $\frac{1}{8}$% discount and less 6% interest for 54 days. If the broker's commission was $\frac{1}{8}$%, what amount did he remit Thompson and what did Thompson make by the transaction ?

8. A wholesale grocer owed for an invoice of $5425.40, purchased in New York, subject to a discount of 6% if paid within 10 days. Within the required time he discounted the bill and remitted for balance as follows: A sight draft which he bought of E. M. Brooks on Gunn & Baker for $4000, at $\frac{1}{2}$% discount, and a bank draft for remainder, the exchange being 10 cents for each $100. How much was required to settle the bill, and how much was gained by discounting it ?

9. Langdon & Perry, of New York, owed on foreign invoices as follows: T. C. Shepherd Sons, London, £1800, 8s.; J. L. Von Buesche, Berlin, 1600 marks; Perrie, Buzzell & Co., Paris, 4016 francs; F. Gonzalez, Mexico, 816 dollars. They bought at their bank: Exchange on London, at 4.86\frac{1}{8}$; on Berlin, $ .238; on Paris, $ .19$\frac{1}{2}$; on Mexico, $ .79$\frac{1}{2}$, and issued one check to cover the total purchase. What was the amount of the check ?

PARTNERSHIP.

566. **Partnership** is the association resulting from an agreement between two or more persons to place their money, effects, labor or skill, or some or all of them, in some enterprise or business, and to divide the profits and bear the losses in certain proportions.

Partnerships may be formed in four different ways, viz.: 1, By written agreement, sealed; 2, by written agreement, unsealed; 3, by oral agreement; and 4, by implication.

Every partnership agreement should be in writing, in which all of the conditions should be fully stated.

567. The **Firm Name** is the style of the concern by which it is legally competent to sue and be sued.

The association is called a firm, house, company or concern.

568. The **Capital** is the money or other property invested.

Usually the investment of each partner is represented in his stock account.

569. The **Resources** or **Assets** of a firm consist of the property owned by it, and of the debts due the firm.

570. The **Liabilities** of a firm are its debts.

571. The **Net Capital** or **Net Worth** of a concern is the amount of the excess of resources over liabilities.

572. The **Net Investment** of a partner is that part of the firm's net capital which he has invested, less the amount which he may have withdrawn from the business.

573. The **Net Gain** or **Net Loss** is the excess of the total gains over the total losses, or the excess of the total losses over the total gains, as the case may be.

574. **Partners** are of four classes: 1. Real or Ostensible Partners, who appear to be, and are, partners in the concern. 2. Dormant or Silent partners, whose names do not appear in the firm name, or whose names are concealed, and yet who profit by the investment. 3. Limited partners, who, according to the requirements of the statute law, publish their connection with the firm, name the limit of their responsibility assumed, and in that manner escape general responsibility. 4. Nominal partners, whose names appear to the public, but who have no investment and receive no share of the gains.

575. To find each partner's share of the gain or loss, and the balance due on settlement, when the gain or loss is divided in proportion to investment.

Example.—Henry P. Long and Thos. E. Bonner were partners in the grocery business. Long invested $1000 in cash, and Bonner invested $2000 worth of goods. Each was to draw $50 per month for services, and the net gain or net loss was to be divided in proportion to each partner's investment. At the end of the year the gain on Mdse. was $2850; the office expenses were $615; there was a private account against Long of $275, and one against Bonner of $410, which amounts were deducted from their salary. Find each partner's share of the gain and the amount due him on final settlement.

OPERATION.

Capital.

Long invested $1000 = $\frac{1}{3}$ of whole.
Bonner invested $2000 = $\frac{2}{3}$ of whole.
Net capital = $3000

Loss and Gain account.

Mdse.,		$2850
Expense,	$615	
Long's salary,	600	
Bonner's salary,	600	$1815
Net gain		= $1035
Long's $\frac{1}{3}$ share,	$345	
Bonner's $\frac{2}{3}$ share,	690	

Due on settlement.

Long, invested,	$1000	
$\frac{1}{3}$ gain,	345	
due on salary,	325	
Present worth,		$1670
Bonner, invested,	$2000	
$\frac{2}{3}$ gain,	690	
due on salary,	190	
Present worth,		$2880

Explanation.—The capital is the sum of the partners' investments, or $3000. Since Long invested $\frac{1000}{3000}$, or $\frac{1}{3}$ of the capital, he is entitled to $\frac{1}{3}$ of the profits. Bonner invested $\frac{2000}{3000}$, or $\frac{2}{3}$ of the capital and is entitled to $\frac{2}{3}$ of the profits. As the total gains are $2850, and the losses, $1815, the net gain is $1035, Long's $\frac{1}{3}$ being $345, and Bonner's $\frac{2}{3}$ being $690. On final settlement there will be due Long as follows: investment, $1000, $\frac{1}{3}$ net gain, $345, balance due on salary ($600—$275) $325, or a total of $1670. There will be due Bonner as follows: investment, $2000, $\frac{2}{3}$ net gain, $690, balance due on salary ($600—$410) $190, or a total of $2880.

Rule.—I. *To divide the loss or gain: Take such a part of the loss or gain as each partner's investment is of the total capital invested.*

II. *To find amount due each partner on settlement: Add to his net investment his net gain and balance due for salary, if any, or deduct from his investment his net loss.*

EXAMPLES.

576. *1.* Maurice Pritchard and Edwin Gaines, of Troy, were partners in the hardware business. Pritchard invested cash, $1800, and Gaines invested cash, $2700. The net gain at the end of the year was $1440, which was divided in proportion to each partner's investment. Find each partner's share of the gain.

2. A, B and C did a piece of work for $1500. A contributed $200 for operating expenses, B, $150, and C, $100. Each was paid a certain sum for his services, and the amount included in the expenses. The net gain was $1050, which by agreement was divided in proportion to the money contributed. Find the share of each.

3. Three men, Jones, Smalley and Parker did a certain piece of work for $4500. Jones worked 25 days, valued at $4 per day, and contributed $575 for operating expenses. Smalley worked 25 days, valued at $5 per day, and contributed $1000 for operating expenses. Parker worked 120 days, valued at $3 per day, and contributed $840 for operating expenses. The money contributed was all expended. If the net gain was divided in proportion to money and services contributed, what was each partner's share?

4. T. W. Guild, L. M. Brooks and James T. Lansing, partners in the firm of Guild, Brooks & Co., have a gain of $12500 to adjust in proportion to each partner's investment. Guild invested cash, $20000 and received $1500 for his services. Brooks invested cash, $30000, and received $1000 for his services. Lansing invested $50000, and made no charge for his services. Find the net gain of each, after deducting the salary account from the gain of $12500, and find the balance due each partner.

5. C. C. Marmaduke, and W. C. Bantling are partners in a general merchandising business. Marmaduke invested a stock of goods valued at $12000, cash $5000, and store and fixtures valued at $4000. Bantling invested cash, $14000. The net gain on merchandise is $18500. The office expenses are $1,500; due Marmaduke for salary, $1200; due Bantling, $1000. Find each partner's share of the gain and his present worth.

577. To find each partner's share of the gain or loss, when divided in proportion to his investment and the time his money was in use.

EXAMPLE.—Edward Bliss and George Blanchard commence business on Jan. 1, 1889, under the firm name of Bliss & Co. Bliss invested $2000 on Jan. 1, 1889, and $3000 additional on Jan. 1, 1890. Blanchard invested $6000 on Jan. 1, 1889, and $8000 on Jan. 1, 1890. On Jan. 1, 1891, their net gain was $2754.81. Find each partner's share of the net gain.

OPERATION.

Bliss' investment.

1st, $2000 for 2 years = $4000 for 1 year.
2nd, = $3000 for 1 year.
Proportionate investment = $7000 for 1 year.

Blanchard's investment.

1st, $6000 for 2 years = $12000 for 1 yr.
2nd, = 8000 for 1 yr.
Proportionate investment = $20000 for 1 yr.

Proportionate capital 1 year = $27000.

Bliss' gain = $\frac{7}{27}$ of $2754.81 = $ 714.21
Blanchard's gain = $\frac{20}{27}$ of $2754.81 = $2040.60
Total gain, $2754.81

EXPLANATION.—Since Bliss' first investment was $2000 for 2 years, and the second, $3000 for 1 year, bringing both to a basis of 1 year, the one of $2000 for 2 years equals the use of 2 × $2000, or $4000 for 1 year, and $4000 for 1 year plus $3000 for 1 year = the proportionate use of $7000 for 1 year. Blanchard's 1st investment of $6000 for 2 years = 2 × $6000, or $12000 for 1 year, and $12000 + $8000 = the proportionate use of $20000 for 1 year. Since the proportionate capital equals the use of $27000 for 1 year, Bliss' share of the gain is $\frac{7000}{27000}$, or $\frac{7}{27}$ of $2754.81 = $714.21; and Blanchard's share of the gain is $\frac{20000}{27000}$, or $\frac{20}{27}$ of $2754.81 = $2040.60.

Rule.—I. *Multiply each investment by the time it was invested, using as a basis or standard of comparison, 1 day, 1 month or 1 year, as the case may be; the product* *will represent the proportionate investment for the unit of time, and the sum of these proportionate investments will represent the proportionate capital for the unit of time.*

II. *Take such a part of the net gain or net loss as each partner's proportionate investment is of the proportionate capital; the quotient will represent his net gain or net loss.*

EXAMPLES.

578. Find each partner's share, based on the following investments and time:

1. A $800 for 9 months, B $2000 for 6 months; gain $400.

2. A $500 for 8 months, B $500 for 10 months, C $500 for 12 months; gain $300.

3. A $500 for 9 months, and $200 for 6 months, B $800 for 9 months; loss $150.50.

4. Moore, $1500 for 1 year, Parks $800 for 18 months; gain $252.

5. Ensign, $950 for 1 year, Gould $800 for 10 months; loss $291.

6. Brown, $500 for 12 months, $300 for 8 months, $200 for 6 months, Smith $800 for 12 months, and $500 for 2 months; gain $292.90.

579. To find each partner's share of the gain or loss and his present worth, when the gain or loss is shared equally, and accounts with losses and gains have been kept, or the resources and liabilities are given.

EXAMPLE.—Benj. Wood and Jas. Cooper formed a partnership, Jan. 10, 1890, under the firm name of Wood & Cooper, for the purpose of conducting a general dry goods business. A memorandum of the articles of copartnership is as follows: Wood is to invest cash, $15000, Cooper, $10000. Each is to draw 6% interest on his investment, and receive a salary of $700 per year for services; they are to share equally in the net loss or net gain of the business. On Jan. 10, 1891, the accounts showing losses and gains were as follows: Mdse., total cost, $98500, sales $85060, unsold $32140. Expenses, total, $10000, except partners salary, and interest due on investment. Resources: Mdse., per inventory, $32140; cash, $5125; notes on hand, $8115; open accounts, $6900. Liabilities: Notes outstanding, $10500; open accounts, $7480; Wood's investment, $15000; Cooper's investment, $10000.

Find each partner's gain and his present worth.

FIRST STATEMENT.

Gains:		
Mdse., total sales,	$85060	
On hand,	32140	
Total,	$117200	
Less cost,	98500	$18700
Losses:		
Expense, total,	10000	
Salary, total,	1400	
Interest, total,	1500	12900
Net gain,		$ 5800

Wood's ½ net gain, $2900.

Cooper's ½ do. do. $2900.

SECOND STATEMENT.

Resources:			
Mdse., on hand,		$32140	
Accts. rec. do.		6900	
Cash in bank,		5125	
Bills rec. on hand,		8115	
Total,			$52280
Liabilities:			
Bills payable, outstanding,		$11100	
Accts. papable, do.		7480	
Total,		$18580	
Wood's investment,	$15000		
Salary,	700		
Interest,	900	$16600	
Cooper's investment,	$10000		
Salary,	700		
Interest,	600	$11300	$46480
Net gain,			$5800

EXPLANATION.—From the total of the sales and inventory, or $117200, subtract the cost, $98500, and the difference, $18700, represents the gain on Mdse. From the gain thus found, subtract the total losses, and the difference, $5800, represents the net gain.

The total resources are $52280. The outside liabilities are $18580. The investment of Wood was $15000. To this add salary for 1 yr., $700, and interest on investment $900, gives $16600, balance of his acct. before the adjustment of the loss and gain acct. The account of Cooper is: original investment, $10000, plus salary, $700, and interest on investment, $600, or a total of $11300. The total due by the concern is: outside liabilities, $18580; Wood's acct. $16600

PRESENT WORTH.

Benjamin Wood's account.			
Invested,		$15000	
Salary,		700	
Interest,		900	
	Total,	$16600	
	Gain,	2900	
	Present worth,		$19500
James Cooper's account.			
Invested,		$10000	
Salary,		700	
Interest,		600	
	Total,	$11300	
	Gain,	2900	
	Present worth,		$14200
	Net capital,		$33700

Total resources,	$52280
Total liabilities,	18580
Net capital,	$33700

and Cooper's acct., $11300 = $46480. This subtracted from the total resources, or $52280 — $46480 = $5800 an increase of resources, or the net gain.

To the net investment of Wood's account of $15000, there has already been added salary, $700, and interest, $900. To the total of $16600 add his ½ net gain, $2900, and the sum, $19500, is his present worth.

Cooper's account, after adding salary, $700, and interest, $600, to the net investment of $10000, is $13000. To this add his gain of $2900, and the sum, $14200, is his present worth.

The sum of Wood's present worth, $19500, and Cooper's present worth, $14200 = $33700, the net capital of the firm. It will be seen that the difference between the total resources and the outside liabilities in the *second* statement ($52280 — $18580 = $33700), is the net capital.

Rule.—I. *To find the net gain or loss: Add to the losses given, the amounts due each partner for salary, and interest, if any, and find the difference between the total losses and gains. Divide the net loss or net gain equally. Or,*

Add to each partner's net investment the amounts due, if any, for services and interest, and include in the statement of resources and liabilities. The difference between the total resources and liabilities will be the net gain.

II. *To find each partner's present worth: To the amount of the net investment, increased by salary, and interest, if any, add his share of the net gain or deduct his share of the net loss. The result will be the present worth of each partner, and the present worth of all the partners will be the net capital of the firm.*

EXAMPLES.

580. *1.* F. G. De Vier and Fred. Van Slyck, partners in the importing firm of De Vier & Van Slyck, have the following losses and gains to adjust: Gain on imported goods, $12875.50; office expenses, $4627.50; due De Vier for 1 year's services, $800; Van Slyck, $1000. Each invested $20000. Find the gain and present worth of each, the gain to be divided equally.

2. On March 1, 1890, Hamlin & Perrin commenced business with a joint capital of $20000, each partner investing one-half. Perrin is to receive a yearly salary of $600 for keeping the books. On March 1, 1891, the total resources were $53500, and the debts of the firm were $28450. Find each partner's share of the net gain and his present worth.

3. At the end of a year the firm of Hayes & McNab finds an increase of net resources amounting to $16450 over joint net investment, $25000 of which was invested by Hayes, and $15000 by McNab. Find the net gain and present worth of each, after allowing 6% on each partner's investment for the year, and Hayes $800 and McNab $1000 for salary, the net gain to be divided equally.

4. J. W. Westlake was associated in business with N. P. Farwell for one year, each investing $5600. At the end of the year their books show the following: Resources: Mdse., $4500; notes, $2500; accounts, $13000; cash, $1500. Liabilities: notes, $5000; accounts, payable $1250; due Farwell for special services, $400. Find the net loss or net gain, which divide equally between the partners, and then find each partner's present worth.

5. Guiness Sons & Co.'s statement shows the following for 1 year:

Debit balances.		Credit balances.	
Cash,	5400.50	Bills payable,	2500
Notes,	16500.50	L. C. Guiness invested	25000
Accounts,	22712.00	J. L. Guiness invested	25000
Real estate,	55500.00	M. N. Porter invested	35000
Total resources,	$100113.00	Investm'ts and debts,	$87500

After allowing each partner 4% interest per annum on his investment, and Porter a salary of $1500 for managing the business, divide the loss and gain equally, and find each partner's present worth.

MISCELLANEOUS EXAMPLES.

581. *1.* Three men jointly own a business block which cost them $150000, and which rents for $16000. A paid $40000 of the purchase price, B, $50000, and C, $60000. How shall the rent be divided?

2. Three men contract to do a certain piece of work and to divide the profits in proportion to the time contributed. A worked 80 days, B, 120 days, and C, 100 days. The net profits were $695.75. Find the share of each.

3. Byron Le Roy and Ralph Huestis are partners. On Jan. 1, 1890, Le Roy invested $8000 and Huestis, $5000. On July 1st, Le Roy withdrew $2000 and Huestis invested $1000. At the end of the year the net loss was $450. Find each partner's share of the loss, it being agreed that the losses or gains should be shared in proportion to the money invested by each and the time it was in the business.

4. The statement of Gorton & Halton on Dec. 31, 1890, shows the following: Mdse. on hand, $1580.25; Mdse. sales, $26332.25; Mdse. purchases, $22,412.50; cash in bank, $5225; cash in safe, $725; notes on hand, $5175; accounts receivable $16819.75; value of store and fixtures, $4500; horses and wagons, $975. The firm owes as follows: On account, $10500; on notes, $4500. Each partner invested $8000 at the beginning of the year. If the total expenses were $1500, what was each partner's share of the gain?

5. Three contractors, Larkin, Pope and Henry, engaged in business for 3 years. Larkin invested $10000, Pope, $12000 and Henry, $15000. At the end of the first year, Larkin drew out $1000, Pope, $2000, and Henry, $500. At the end of the second year Larkin drew out $2000, Pope, $1000, and Henry, $2000. At the end of the third year, there were no liabilities, and the net assets were $45600. If they agreed to share the net gain in proportion to averge investment, what was each partner's present worth at the end of the third year?

6. The following is a statement of the business of Allen & Furbeck on June 1, 1891, for the year just closed: *Resources:* Merchandise on hand, $5854.90; notes on hand, $6855.10; accounts due the firm, $3128.50; cash on hand and in bank, $8228.50. *Liabilities:* Notes outstanding, $3500; accounts due by the firm, $11275.10; due for 3 mos. rent, to C. D. Grant, $225; due Allen for 1 year's services, $1200; due Furbeck for 1 year's services, $900. If Allen invested $2500 in cash, and Furbeck, $3500 in cash, what is the net gain and what is the present worth of each, there being no interest allowed on investment, and the net loss or net gain being shared equally?

7. Theodore Blanchard, a contractor, and one of his foremen, agreed to build a house for James Barry, and to share in the profits, under the following conditions: The foreman was to contribute $2000 in cash, and his services, valued at $5 per day. Blanchard was to contribute $9600 in cash. The profits were to be divided in proportion to the money invested and the value of the services rendered. The foreman worked 80 full days. At settlement it was found that the profits, after allowing for the foreman's services, were $3300. Find each partner's share of the profits.

8. H. L. McDowell and his son, J. W. McDowell, formed a copartnership for 1 year. H. L. McDowell invested $10000, but his son had no capital to invest. It was agreed that the father should be paid 6% interest on his investment and give no time to the business, that the son should be paid a yearly salary of $500, and that the net gain should be divided equally; but if a loss, the son's loss should not exceed his salary earned, which shall remain in the business. At the end of the year the gain on Mdse. was $2255.50; the office expenses, not including salary and interest, were $805.50. Find the net gain and present worth of each.

9. L. M. Sprague and F. O. Mareniss are partners. Sprague invested $4500 in cash and Mareniss contributed merchandise to the value of $10850. They agreed that each partner should have 6% interest on his investment, that all withdrawals not exceeding $50 per month for services should be charged to private account of the partner withdrawing it and the net loss or net gain should be divided equally. A statement of their losses and gains at the end of the year is as follows: Gain on merchandise, \$14212.50; expenses, not including salary of \$600 for each partner and interest on each investment \$4591.50. Sprague has charged in private account \$250, and Mareniss, \$412. After adjusting salaries and interest on investment, what is each partners net gain, amount due on private account and his present worth ?

10. James T. Morris, Levi D. Perkins and Frank C. Livingston are partners in the dry goods business, under the firm name of Morris, Perkins & Co. On commencing business, Jan. 1, 1890, Morris invested \$4000, Perkins, \$5000, and Livingston, \$6000. On Dec. 31, 1890, their bookkeeper rendered a statement as follows: Mdse., cost of purchases, \$85514.20; sales, \$91612.80; value of the unsold, \$5211.27. Real estate, first cost, \$25000; cost of repairs, \$1400; returns for rent, \$1850; inventory value, \$28000. Cash, on hand, \$212.75; in bank, \$1275.25. Notes, on hand, \$6500. Accounts due firm, \$11517.98. Interest accrued on accounts due the firm, \$412.50. Cost and present value of horses and wagons, \$1150. Office expenses for the year, \$2250.10. Mortgage on the firm's real estate, \$10000. Interest accrued on mortgage, \$600. Notes outstanding, \$5500. Accounts owing by the firm, \$10857.48. There is due the partners, and not yet entered in their accounts or charged in the expenses of the business, 6% interest on their investments for one year, and also for services as follows: Morris, \$1800; Perkins, \$1500, and Livingston, \$1000. Determine the net loss or the net gain and divide it equally among the partners. Prove the result by a statement of resources and liabilities, and compare it with a detailed statement of losses and gains. Show in each statement the present worth of each partner, and the net capital of the concern.

ANSWERS.

Page 8.

Art. 36.

1. 5543.
2. 4377.
3. 4557.
4. 4850.
5. 5440.
6. 5322.
7. 50366.
8. 53248.
9. 51835.
10. 51856.
11. 47913.
12. 53195.
13. 372105.
14. 421302.
15. 368895.
16. 320180.
17. 344051.
18. 354969.

Page 9.

19. 109120.
20. 69310.
21. 1436076.
22. 498951.
23. 341207.
24. 302991.

Page 11.

Art. 40.

1. 534.
2. 688.
3. 870.
4. 3064.
5. 4731.
6. 3164.
7. 3479.
8. 5582.
9. 10003.
10. 528058,
11. 370 bushels.
12. 3165 pounds.

Page 12.

13. 425 bushels.
14. 8353 persons.
15. 1748 yds.
16. 1460624 sq. miles.

Page 13.

Art. 42.

1. 10617.42.
2. 3961.40.
3. 3717.97.
4. 19050.06.
5. 6486.39.
6. 81987.77.
7. 24995.82.
8. 830883.42.
9. 284128.17.
10. 965682.22.

Page 15.

Art. 46.

1. 92.
2. 397.
3. 384.
4. 3739.
5. 8546.
6. 611^2.
7. 433^3.
8. 575.
9. 719.
10. 923.

Art. 47.

1. 288.71.
2. 493.98.
3. 573.57.
4. 527.61.
5. 736.83.

Page 17.

Art. 54.

1. 253.
2. 169.
3. 474.
4. 356.
5. 86.
6. 5457.
7. 6298.
8. 1985.
9. 3712.
10. 7001.
11. 3067.
12. 1009.
13. 3645.
14. 7655.
15. 8889.
16. 512 lbs. Clark.
17. $834 on deposit.
18. 5 times.
19. 12960 acres rem.
20. $1984 gain.
21. $140 amt. left.
22. $7520 face of note.

Page 18.

23.
Jan.
1. $750.50.
5. $751.
10. $836.
15. $778.50.
20. $785.50.
25. $933.
28. $981.75.
30. $984.75.
31. $975.25.
24.
Mar.
1. $655.
5. $730.
10. $1121.17.
15. $1061.81.
20. $836.49.
31. $856.31 balance in bank.

Page 21.

Art. 61.

1. 2650.
2. 23188.
3. 7665.
4. 3516.
5. 23413.
6. 9576.
7. 12976.
8. 14418.
9. 42084.
10. 1743
11. 4152.
12. 12342.
13. 8333.
14. 35610.
15. 7872.
16. 27120.
17. 6454.
18. 53130.

Art. 63.

1. 302418.
2. 205683.
3. 2151090.
4. 1091510.
5. 887112.
6. 135676.
7. 206010.
8. 239018.
9. 6220848.
10. 389675.
11. 767602.
12. 576450.
13. 295500.
14. 109536.
15. 499010.
16. $55125.

Page 22.

17. $381615 sales.
18. $189739175.
19. 1945710 pop.
20. $734803.

Page 23.

Art. 65.

1. 4800.
2. 105000.
3. 20800.
4. 108000.

5. 338000.
6. 1134000.
7. 2160000.
8. 216000.
9. 5358400.
10. 10800000.
11. 30386700.
12. 68162400.
13. 632415000.
14. 542088800.
15. 4191453000.

Page 24.

Art. 68.

1. 25683.
2. 56790.
3. 19817.
4. 142817.
5. 30630.
6. 329827.
7. 26585.
8. 150810.
9. 3471876.
10. 360072.
11. 6755283.
12. 6135450.
13. 14407857.
14. 23737075.
15. 13827030.

Art. 69.

1. $463.26.
2. $7358.26.
3. $62.86.
4. $279.67.
5. $2182.
6. $1279.16.
7. $925.49.
8. $185.25.

Page 26.

Art. 79.

1. 815.
2. 281.
3. 441.
4. $3945\frac{2}{3}$.
5. $1721\frac{1}{13}$.
6. 12870.
7. 4637.
8. $1813\frac{5}{8}$.
9. $4056\frac{2}{13}$.
10. $18559\frac{2}{11}$.
11. $4493\frac{4}{13}$.
12. $5789\frac{6}{14}$.
13. $1361\frac{1}{13}$.
14. $7063\frac{8}{13}$.
15. $13023\frac{5}{11}$.

Page 27.

Art. 81.

1. $11\frac{64}{100}$.
2. $89\frac{11}{80}$.
3. $150\frac{49}{60}$.
4. $832\frac{51}{100}$.
5. $83\frac{125}{500}$.
6. $134\frac{475}{700}$.
7. $66\frac{424}{1100}$.
8. $422\frac{3}{120}$.
9. $155\frac{24}{400}$.
10. $24\frac{463}{500}$.
11. 125.
12. $138\frac{489}{900}$.
13. $776\frac{109}{800}$.
14. $301\frac{1009}{8000}$.
15. 4.
16. $543\frac{13}{40}$ acres.
17. $857\frac{13}{80}$ acres.

Page 29.

Art. 83.

1. 36.
2. 15.
3. 5.
4. 12.
5. $50\frac{13}{45}$.
6. $3903\frac{11}{23}$.
7. $2761\frac{8}{19}$.
8. $12\frac{255}{367}$.
9. $315\frac{205}{793}$.
10. $2823\frac{11}{31}$.
11. $746\frac{50}{153}$.
12. $117\frac{43}{251}$.
13. $309\frac{1050}{2154}$.
14. $61286\frac{857}{1525}$.
15. $265095\frac{1140}{1888}$.
16. $3960\frac{4289}{5407}$.
17. $11471\frac{721}{999}$.
18. $7837\frac{3503}{11181}$.
19. 4025 elephants.
20. $173\frac{50}{280}$ av. miles.
21. $\$676605\frac{2061}{2518}$ average cost per mile.
22. $86781\frac{50868}{62525}$ miles.
23. $\$17907\frac{225}{785}$ capital.
24. $\$1506849\frac{115}{365}$ expts. average.
25. $\$2,906,070,732\frac{3}{13}$ New York.
$\$1,437,821,308\frac{6}{13}$ remaining cities.

Page 32.

Art. 100.

1. 600¢.
2. 11100¢.
3. $2.41.
4. $10.44.
5. $214.68.
6. $18.
7. $510.
8. $98.76.
9. 10098¢.
10. 375¢.
11. 2653¢.
12. 15732¢.

Page 33.

Art. 102.

1. $88.84.
2. $144.89.
3. $4337.77.
4. $220.93.
5. $378.07.
6. $377.29.
7. $384.51.
8. $1088.60.
9. $4073.76.
10. $896.86.

Page 34.

Art. 104.

1. $8162.75.
2. $4859.76.
3. $27.36.
4. $792.
5. $43.82.

Page 35.

Art. 106.

1. $265.
2. $1786\frac{29}{44}$.
3. 3812.
4. $107.10.
5. $3437\frac{5}{33}$.
6. $15.50.
7. 142.
8. $18181\frac{45}{55}$.
9. $157.87.
10. $4060\frac{200}{293}$.
11. $38.88.
12. 15680.
13. $20.22.
14. $146\frac{203}{216}$.
15. $58557\frac{375}{1135}$.
16. $2098.24.
17. $1578.75.
18. 950 bu.

Page 39.

Art. 114.

1. $22.95. *10.* $12.
2. $37.75. *11.* $26.40.

3. $2.25. *12.* $9.90.
4. $1.56. *13.* $8.38.
5. $1.50. *14.* $9.63.
6. 25¢. *15.* $8.48.
7. 96¢. *16.* $9.87.
8. $1.20. *17.* $11.55.
9. $1. *18.* $140.

Page 40.

Art. 117.

1. $8.33. *6.* $4.13.
2. $2.04. *7.* $24.
3. $3.36. *8.* $3.84.
4. $240. *9.* $120.
5. $10.64. *10.* $270.

Page 41.

Art. 120.

1. $17. *6.* $136.65.
2. 20¢. *7.* $2.40.
3. $4.88. *8.* $14.
4. 18¢. *9.* $13.50.
5. $2.77. *10.* $2.25.

Page 42.

Art. 122.

1. $1.25.
2. $5.53.
3. $1.28.
4. $172.26.
5. $13.79.
6. 1¢.
7. 3¢.
8. 61¢.
9. $25.97.
10. $662.32.

Page 43.

Art. 125.

1. $9.32. *6.* 45¢.
2. $8.74. *7.* $26.
3. $33.18. *8.* $45.
4. $1.04. *9.* $27.13.
5. $1.70. *10.* $2.35.

Art. 126.

1. $4.38. *11.* $2.34.
2. $4.50. *12.* $6.88.
3. $3.71. *13.* $6.32.
4. $13.95. *14.* $3.60.
5. $6.93. *15.* $16.39.
6. 75¢. *16.* $1.15.
7. $3.50. *17.* $3.74.
8. $4. *18.* $7.77.
9. $2.34. *19.* $1.34.
10. $11.40. *20.* $8.22.

21. $8.13.
22. $608.70 acct. and int.
23. $3 int.
24. $15.40 cash.
25. $7.66 discount.
$930.46 cash.

Page 45.

Art. 133.

1. 3^3.
2. 13, 3^2.
3. 3, 5, 11.
4. 3, 31.
5. 3^2, 2^3, 11.
6. 2^2, 3^3.
7. 2^2, 43.
8. 3^2, 5, 2^2.
9. 2^3, 5, 3^2.
10. 2^9.
11. 5^2, 7, 3, 2.
12. 2^4, 3^2.
13. 5^6.
14. 13, 3^3, 2^6.
15. 5, 2, 7^3, 53.

Page 47.

Art. 139.

1. 19. *6.* 36.
2. 17. *7.* 4.
3. 13. *8.* 21.
4. 125. *9.* 4.
5. 144. *10.* 150.

Page 48.

Art. 145.

1. 352. *8.* 360.
2. 1020. *9.* 60.
3. 285. *10.* 1200.
4. 1053. *11.* 8640.
5. 900. *12.* 3584.
6. 660. *13.* 330.
7. 2200. *14.* 630.
15. 1078.

Page 49.

Art. 148.

1. 30.
2. $192\frac{18}{31}$.
3. 90.
4. $97\frac{1}{3}$.
5. $86\frac{54}{135}$.
6. 105 chickens.
7. 45 bu. potatoes.
8. 38 bales and 528 odd yards.

Page 50.

Art. 154.

1. $\frac{4}{15}$. *7.* $\frac{88}{99}$.
2. $\frac{17}{19}$. *8.* $\frac{19}{20}$.
3. $\frac{6}{7}$. *9.* $\frac{45}{48}$.
4. $\frac{2}{13}$. *10.* $\frac{14}{60}$.
5. $\frac{6}{100}$. *11.* $\frac{18}{15}$.
6. $\frac{17}{40}$. *12.* $\frac{42}{80}$.
13. $\frac{2}{21}$.

Page 51.

Art. 162.

1. $\frac{10}{15}$. *8.* $\frac{9}{27}$.
2. $\frac{3}{15}$. *9.* $\frac{9}{120}$.
3. $\frac{6}{21}$. *10.* $\frac{55}{88}$.
4. $\frac{14}{21}$. *11.* $\frac{88}{64}$.
5. $\frac{15}{36}$. *12.* $\frac{10}{115}$.
6. $\frac{12}{36}$. *13.* $\frac{24}{125}$.
7. $\frac{15}{36}$. *14.* $\frac{40}{195}$.
15. $\frac{77}{147}$.

Page 52.

Art. 164.

1. $\frac{2}{3}$. *9.* $\frac{3}{40}$.
2. $\frac{1}{5}$. *10.* $\frac{7}{8}$.
3. $\frac{3}{7}$. *11.* $\frac{7}{13}$.
4. $\frac{5}{7}$. *12.* $\frac{3}{35}$.
5. $\frac{5}{13}$. *13.* $\frac{3}{16}$.
6. $\frac{4}{13}$. *14.* $\frac{5}{24}$.
7. $\frac{5}{30}$. *15.* $\frac{11}{24}$.
8. $\frac{3}{5}$. *16.* $\frac{31}{60}$.

Page 53.

Art. 166.

1. $\frac{5}{8}$. *7.* $\frac{2}{3}$.
2. $\frac{1}{10}$. *8.* $\frac{17}{24}$.
3. $\frac{3}{5}$. *9.* $\frac{25}{48}$.
4. $\frac{3}{5}$. *10.* $\frac{3}{5}$.
5. $\frac{4}{5}$. *11.* $\frac{1}{12}$.
6. $\frac{5}{7}$. *12.* $\frac{3}{5}$.

Art. 168.

1. \$$\frac{87}{7}$.
2. $\frac{85}{4}$ hrs.
3. $\frac{11}{3}$ tons.
4. $\frac{50}{3}$¢.
5. $\frac{103}{3}$ #.
6. $\frac{372}{5}$ cwt.
7. $\frac{152}{5}$ M.
8. $\frac{20747}{12}$.

Page 54.

Art. 170.

1. $7.
2. $9\frac{10}{13}$ yds.
3. $3\frac{10}{33}$ lbs.
4. $33\frac{1}{3}$¢

5. $3\frac{75}{125}$ cds.
6. 12 cu. feet.
7. $8\frac{85}{225}$ cwt.
8. $8\frac{4}{126}$ tons.
9. $5\frac{104}{150}$ M ft. lumber.
10. $15\frac{20}{125}$ inches.

Art. 172.

1. $\frac{56}{84}$, $\frac{63}{84}$, $\frac{60}{84}$.
2. $\frac{45}{90}$, $\frac{20}{90}$, $\frac{54}{90}$.
3. $\frac{210}{280}$, $\frac{224}{280}$, $\frac{140}{280}$, $\frac{200}{280}$.
4. $\frac{1020}{2400}$, $\frac{1680}{2400}$, $\frac{1200}{2400}$, $\frac{800}{2400}$.
5. $\frac{120}{720}$, $\frac{144}{720}$, $\frac{180}{720}$, $\frac{240}{720}$, $\frac{360}{720}$.
6. $3\frac{18}{24}$, $5\frac{16}{24}$, $14\frac{12}{24}$.
7. $17\frac{27}{36}$, $8\frac{12}{36}$, $6\frac{24}{36}$.
8. $21\frac{60}{180}$, $16\frac{36}{180}$, $19\frac{90}{180}$.
9. $8\frac{60}{120}$, $9\frac{40}{120}$, $10\frac{30}{120}$, $11\frac{24}{120}$.
10. $175\frac{18}{30}$, $75\frac{15}{30}$, $100\frac{20}{30}$.

Page 55.

Art. 174.

1. $\frac{8}{60}$, $\frac{47}{60}$, $\frac{50}{60}$, $\frac{34}{60}$.
2. $\frac{22}{96}$, $\frac{5}{96}$, $\frac{56}{96}$, $\frac{60}{96}$.
3. $\frac{36}{81}$, $\frac{24}{81}$, $\frac{17}{81}$, $\frac{5}{81}$.
4. $\frac{20}{144}$, $\frac{17}{144}$, $\frac{70}{144}$, $\frac{104}{144}$.
5. $\frac{45}{150}$, $\frac{121}{150}$, $\frac{126}{150}$, $\frac{180}{150}$.
6. $\frac{258}{512}$, $\frac{253}{512}$, $\frac{276}{512}$.
7. $8\frac{24}{32}$, $7\frac{10}{32}$, $3\frac{1}{32}$.
8. $13\frac{8}{12}$, $17\frac{6}{12}$, $19\frac{9}{12}$.
9. $12\frac{6}{15}$, $14\frac{10}{15}$, $18\frac{9}{15}$.
10. $\frac{720}{1728}$, $\frac{888}{1728}$, $\frac{1158}{1728}$, $\frac{1116}{1728}$.

Art. 176.

1. $1\frac{1}{18}$. 4. $\frac{146}{150}$.
2. $1\frac{11}{20}$. 5. 2.
3. $2\frac{7}{25}$. 6 $2\frac{1}{144}$.

Page 56.

Art. 178.

1. $2\frac{18}{60}$. 8. $1\frac{1}{6}$.
2. $1\frac{27}{40}$. 9. $\frac{5}{6}$.
3. $1\frac{82}{105}$. 10. $1\frac{11}{32}$.
4. $1\frac{59}{72}$. 11. $1\frac{1}{95}$.
5. $\frac{81}{80}$. 12. $\frac{81}{890}$.
6. $2\frac{37}{81}$. 13. $2\frac{625}{1886}$.
7. $2\frac{19}{144}$. 14. $1\frac{9}{14}$.
15. $1\frac{9}{64}$.

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Art. 180.

1. $846\frac{369}{616}$.
2. $143\frac{9}{10}$.
3. $185\frac{111}{1809}$.
4. $155\frac{17}{180}$.
5. $321\frac{2}{3}$.
6. $388\frac{29}{40}$.
7. $109\frac{23}{32}$ Acres in farm.
8. $82\frac{31}{36}$ tons coal.
9. $208\frac{7}{8}$ yards.

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10. $621\frac{247}{800}$ tons. \$2992.$\frac{19}{20}$.

Art. 182.

1. $\frac{1}{2}$. 6. $1\frac{1}{120}$.
2. $\frac{1}{12}$. 7. $\frac{3}{8}$.
3. $\frac{3}{35}$. 8. $\frac{1}{18}$.
4. $\frac{7}{16}$. 9. $\frac{13}{60}$.
5. $\frac{1}{56}$. 10. $\frac{11}{25}$.

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Art. 184.

1. $9\frac{1}{4}$. 6. $32\frac{7}{12}$.
2. $63\frac{5}{8}$. 7. $63\frac{2}{5}$.
3. $136\frac{2}{3}$. 8. $54\frac{28}{45}$.
4. $93\frac{3}{8}$. 9. $55\frac{6}{11}$.
5. $10\frac{31}{60}$. 10. $93\frac{13}{17}$.

Art. 186.

1. $1\frac{7}{8}$. 6. $2\frac{1}{2}$.
2. $2\frac{1}{2}$. 7. $5\frac{2}{3}$.
3. $3\frac{3}{19}$. 8. $3\frac{1}{2}$.
4. $2\frac{1}{5}$. 9. $7\frac{5}{12}$.
5. $3\frac{1}{16}$. 10. $1\frac{4}{11}$.

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Art. 188.

1. 4. 6. 36.
2. $2\frac{5}{8}$. 7. 12.
3. 9. 8: 35.
4. $1\frac{16}{35}$. 9 35.
5. $7\frac{5}{7}$. 10. $\frac{7}{11}$.

Art. 190.

1. $\frac{9}{25}$. 5. $\frac{19}{186}$.
2. $\frac{3}{7}$. 6. $\frac{16}{75}$.
3. $\frac{2}{11}$. 7. $\frac{3}{10}$.
4. $\frac{3}{25}$. 8. $\frac{2}{9}$.
9. $\frac{6}{13}$.

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Art. 192.

1. \$$720\frac{3}{8}$.
2. \$$429\frac{8}{15}$.
3. \$165.
4. \$$21\frac{21}{50}$.
5. \$$3840\frac{41}{90}$.

Page 61.

Art. 194.

1. $\frac{2}{41}$. 3. $\frac{17}{851}$.
2. $\frac{5}{61}$. 4. $\frac{5}{91}$.
5. $25\frac{1}{10}$. 9. $34\frac{11}{36}$.
6. $5\frac{11}{225}$. 10. $21\frac{2}{3}$
7. $12\frac{1}{17}$. 11. $144\frac{13}{144}$.
8. $20\frac{50}{81}$. 12. $278\frac{2}{3}$.

Page 62.

Art. 196.

1. $39\frac{2}{3}$. 8. $14\frac{2}{5}$.
2. $19\frac{4}{5}$. 9. 36.
3. $21\frac{3}{7}$. 10. 21
4. $51\frac{3}{5}$. 11. 20.
5. $888\frac{4}{5}$. 12. $22\frac{2}{9}$.
6. 40. 13. $67\frac{8}{25}$.
7. 300. 14. $53\frac{2}{11}$.
15. $3\frac{22}{51}$.

Art. 198.

1. $\frac{981}{2250}$. 7. $\frac{57}{85}$.
2. $6\frac{3}{20}$. 8. $\frac{3}{5}$.
3. 1. 9. $\frac{5}{12}$.
4. $\frac{19}{108}$. 10. $\frac{2}{3}$.
5. 18. 11. $3\frac{3}{35}$.
6. $2\frac{1}{3}$. 12. $3\frac{45}{88}$.

Art. 199.

1. $7\frac{29}{132}$.
2. $1\frac{59}{117}$.
3. $\frac{4}{35}$, \$$441\frac{3}{5}$ Emerson's.
4. $55.01\frac{47}{63}$.
5. 30 ds.

Page 63.

6. $5\frac{48}{170}$ hrs.
7. \$1430, wife; \$357.50, 1st son; \$715, 2nd son; \$1072.50, 3d son.
8. \$$108\frac{36}{133}$, M. \$$129\frac{128}{133}$ S. \$$121\frac{107}{133}$ C.
9. \$62.50 cost.
10 \$15 sell. price.
11. 15 suits.
12. 2966 bu.

Page 66.

Art. 211.

1. .026. 4. .04.
2. .27. 5. .00022.
3. .0016. 6. 5.7.

Page 67.

7. 83.0504.
8. 710.00243.
9. 500.05.
10. 45.046.

Art. 212.

1. 30.75. 2. 30.75.

3. 8.25.
4. 8.25
5. 5.63.
6. 3.815.
7. 3.815.
8. 5.08.
9. 5.08.
10. 1.001.
11. 10.007.
12. 15.0015.
13. 51.7, 5.17, .517.
14. 350.6, 35.06, 3.506, .3506, .03506.

Art. 214.

1. $\frac{3}{10}$.
2. $\frac{4}{5}$.
3 $\frac{1}{4}$.
4. $\frac{3}{4}$.
5. $\frac{3}{8}$.
6. $\frac{5}{8}$.
7. $\frac{7}{8}$.
8. $\frac{57}{200}$.
9. $\frac{366}{625}$.
10. $16\frac{3}{4}$.
11. $125\frac{12}{25}$.
12. $24\frac{6}{25}$.
13. $\frac{1}{8}$.
14. $\frac{5}{8}$.
15. $\frac{3}{8}$.
16. $\frac{53}{160}$.
17. $\frac{21}{25}$.
18. $\frac{101}{4000}$.

Page 68.

Art. 216.

1. .5.
2. .75.
3. .8.
4. .625.
5. .875.
6. $.5555\frac{5}{9}$.
7. .3125.
8. $.3333\frac{1}{3}$.
9. $.6666\frac{2}{3}$.
10. .0625.
11. $.4166\frac{2}{3}$.
12. .226.
13. .76.
14. .015625.
15. .00024.

Page 69.

Art. 218.

1. 309.80565.
2. 151.82 lbs.
3. $884.5.
4. $809.8825.
5. 41.885 cords.
6. 54.6 ozs.
7. 136.33 M ft. lumber.
8. 102.983 tons coal.
9. $417\frac{4877}{5000}$.
10. 103.183 M. $1403.84.

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Art. 220.

1. .628.
2. .66.
3. .908.
4. 6.904 brls.
5. 100.5 M ft. lumber.
6. $10.97.
7. $90.855.
8. $96.355.

Page 71.

Art. 222.

1. .375.
2. .136.
3. .0744.
4. .01988.
5. .30875.
6. 15.066.
7. 136.05.
8. .73
9. $.87033\frac{1}{3}$.
10. $118.6896.
11. $\$3.1533\frac{1}{3}$.
12. 5.7431.
13. $3510.71.
14. $1669.33.

Page 72.

Art. 225.

1. 250.
2. 25.
3. 12.
4. 1200.
5. .5.
6. .002.
7. .00002.
8. 9.
9. .01.
10. .05.
11. 20.
12. $.0566\frac{2}{3}$.

Page 73.

13. 211.11.
14. 20202.20020002.
15. 50500605.
16. 1322.11.
17. 500200.040224.

Art. 226.

1. $1100.87.
2. 1011.6 cu. yds.
3. $7290.
4. $37.50.
5. $31.75.
6. 25.036 M ft.

Page 74.

7. C. $6480, B. & S. $2025, G. $2295, Notions, $2700.
8. $6683.75.
9. B. 360, S. 60, T. 15, E. 45.
10. $3457.20.

Page 82.

Art. 251.

1. 9624 grs.
2. 142 ozs.
3. 40 qts.
4. $36\frac{1}{2}$ gals.
5. 593 gi.
6. 5285 ft.
7. 46282.5 sq. ft.
8. 22400 lbs.
9. 55296 cu. in.
10. 9960 hrs.
11. 1 pk., 3 qts., 1 pt.
12. 1.6 pks.
13. 142 cu. ft.
14. 1474 far.
15 $1\frac{32}{45}$ pt.
16. 35000 T. grs.
17. 1 pk., 1 qt., 1 pt.
18. 1 qt., $\frac{1}{2}$ pt.
19 8295 links.
20. 7510 grs.

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Art. 253.

1 16 gal., 1 pt.
2. 1 lb., 2 ozs., 11 pwt., 16 grs.
3 5 gals., 3 qts., 1 gi.
4. 3 cds., 16 cu. ft.
5. £1, 3 far.
6. $17, 8d, 5¢, 2 m.
7. 6 A. 20 sq. rds.
8. 10 lbs., 8 ozs.
9. 7 sq. yds., 3 sq. ft., 40 sq. in.
10. 22 ch. 2 rds. 4 lks. 2.32 in.
11. $1\frac{3}{4}$ qts.
12. $\frac{2}{3}$ lb.
13. $.039\frac{1}{16}$ lbs.
14. .09375.
15. $36\frac{1}{4}$ brls.

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Art. 255.

1. 6 bu. 6 qts. 1 pt.
2. 6 lbs. 1 oz. 8 pwts. 14 grs.
3. 6 yrs. 5 mos. 18 days, 18 hrs. 6 min. 20 secs.
4. 6 cds. 4 cd. ft. 12 cu. ft.
5. 8 mi. 122 rds. 1 yd. 2 ft. 1 in.
6. 15 T. 11 cwt. 1 qr. 22 lbs.
7. 7 A. 30 sq. rds. 8 sq. ft. 40 sq. in.
8. £12, 12s, 7d.
9. 8 brls. 14 gals.
10. $59, 5d, 9¢, 4 m.

Art. 257.

1. 17 lbs. 8 oz. 19 pwts. 19 grs.
2. 16 bu. 1 pt.
3. 4 cds. 10 cu. ft.
4. 1 brl. 30 gals. 1 qt. 1 pt.
5. £13, 19s, 11d, 1 far.
6. 5 mi. 228 rds. 8 ft. 6 in.
7. 56 sq. rds.
8. 16 T. 14 cwt. 1 qr.
9. 8 ds. 18 hrs. 52 min.
10. $17, 5d, 8¢, 2m.

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Art. 259.

1. 76 bu. 2 pks. 4 qts. 1 pt.
2. 8 mi. 20 rds. 1 yd. 1 ft.
3. 9 brls. 2 gals. 2 qts.
4. 17 A. 64 rds.
5. 144 cds.
6. £54, 15s, 4d.
7. 136 lbs. 1 pwt.
8. 51 T. 5 cwt. 1 qr 3 lbs.
9. 135 yrs. 9 hrs. 58 min. 30 secs.
10. $187, 5d., 8¢.

Art. 261.

1. 3 A. 20 sq. rds. 15 sq. yds. 1 sq. ft. 18 sq. in.
2. 5 mi. 201 rds. 1 ft. $\frac{3}{8}$ in.
3. 2 bu. 3 pks.
4. 2 cds. $11\frac{1}{5}$ cu. ft.
5. 13 cwt.
6. 1 yr. 274 ds. 7 hrs. 15 min.
7. 3 lbs. 4 oz. 13 pwts. 8 grs.
8. £3, 14s, $3\frac{1}{5}$ far.
9. 12 brls. 16 gals. 2 qts. $\frac{3}{5}$ pt.
10. 5 bu. 3 pks. 7 qts. $\frac{6}{7}$ pt.

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Art. 274.

1. 78 sq. ft.
2. $66\frac{2}{3}$ sq. yds.
3. $32\frac{2}{5}$ A.

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4. 360 A.
5. 259.182 + feet.
6. 28.64 + rds.
7. $37.70.
8. 26649.9072 gals.
9. 45 sq. ft.
10. $26\frac{2}{3}$ sq. yds.
11. 201.0624 sq. in.
12. 33.5104 cu. feet.
13. 117.6264 gals.

Art. 275.

1. £380, 6s, 1d.
2. $79.93.
3. 14 yr. 112 ds.
4. 93 T. 11 cwt. 38 lbs.
5. 1235 lbs., 6 oz. 13 pwt. $20\frac{1}{2}$ grs.

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6. 41.26 + brls.
7. 180.6 + rds.
8. 340.86 + sq. ft. lead.
9. 113989.6 + francs.
10. 8 T. 2 cwt.
11. 1876.857 + bush.
12. 442.9 + ft. length.
13. 6.076 + lbs. Troy.
14. $359.38 gain.
15. $742.
16. $50.92.
17. 980 T. 12 cwt. 32 lbs. $4167.62.
18. $1279.80.
19. $4299.80. $4338.30.
20. 2 mi. 108 rds. 6 ft.

Page 91.

Art. 279.

1. $547.68.
2. $498.61.

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3. $1044.83.
4. 1131.77.
5. $468.60.
6. $567.52.

Page 93.

Art. 281.

1. $4153.38.
2. $1198.75.
3. $4277.63.
4. $3584.00.

Page 99.

Art. 297.

1. $494\frac{6}{11}$ perches.
2. 648 cu. feet.
3. $284.28.
4. 85 ft. 1 in. +
5. $90.84.

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6. $997\frac{1}{3}$ ft. lumber.
7. 31 days.
8. 192 gals.
9. 85 board ft.
10. $\frac{4}{5}$ ton.
11. $1253\frac{3}{11}$ perches.
12. 10 mos. 29 days; 333 days.
13. 328556 + bricks.
14. 112 yds.
15. 12 T. 80 lbs.
16. 12 brls.
17. 914 days.
18. $1316\frac{1}{4}$ cu. ft.
19. 44 T. 2 cwt. 10 lb.; 44.105 T.
20. 64977 + bricks.
21. 45 yds. $61\frac{1}{2}$ yds.

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22. 1948 ds.; 5 yrs. 4 mos. 2 ds.
23. 292020 bricks.
24. $127.05.
25. 216 cu. yds.
26. 15931 bricks.
27. 18 board ft.
28. $20.13.
29. 40.68 cu. feet.
30. 655 ds.
31. $58.97.
32. $2552.52.
33. $89.10.
34. $51\frac{3}{4}$ yds. $75.04

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35. $496.36.
36. $191.03.
37. $3.00.
38. 2926 ft.
39. 3552 ft.
40. $53.28.

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Art. 316.

1. 5 yrs.
2. $450.

3. 4000 lbs.
4. 150 bu.
5. 28000 ¢.
6. 48 M ft.
7. 1236 pwts.
8. 4 days.
9. 390 bricks.
10. $1125.50.

Art. 317.

1. 4 lbs.
2. $15.10.
3. 250 men.
4. 200 ozs.
5. 120¢.
6. 54 days.
7. 588.75 cords.
8. 121 ft.
9. 36 miles.
10. $135.09.
11. $500.
12. 1500 Germans, 2500 Eng., 375 French, 100 Italians, 8025 Americans.

Page 106.

13. 150 sheep.
14. 875 T. grate, 187$\frac{1}{2}$ T. egg, 250 T. nut, 150 T. pea, 287$\frac{1}{2}$ T. No. 4.
15. 603$\frac{1}{3}$ Acres.

Art. 319.

1. 1700. 3. $1500.
2. 700. 4. 400 lbs.
5. 1500 ozs.

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6. $1200.
7. 4$\frac{1}{2}$.
8. $\frac{4}{7}$.
9. 75%.
10. 66$\frac{2}{3}$%.
11. $4500.
12. 1000 bu.
13. 63 gals. 57.96 gals.
14. 584$\frac{3}{8}$ bu.
15. $251.67.

Page 108.

Art. 321.

1. 20%. 6. 5%.
2. 10%. 7. 2$\frac{2}{3}$%.
3. 2%. 8. 10%.
4. 5%. 9. 10%.
5. 8$\frac{1}{3}$%. 10. $\frac{1}{4}$%.

11. 15%. 13. 75%.
12. 20%. 14. 9%.
15. 2$\frac{1}{4}$%.

Page 109.

Art. 323.

1. 277.21. 3. 840.
2. 2580. 4. 637.
5. 250.
6. 495 sheep.
7. 19200 pop.
8. 540 children.
9. 840 trees.
10. $2242.

Page 110.

Art. 325.

1. 680.
2. 12 chickens.
3. 500.
4. $36855.90.
5. 8750 bu.

Art. 327.

1. 200. 2. $660.

Page 111.

3. $2000.
4. $1125.00.
5. $700.90.

Art. 328.

1. 20%. 4. 25% profit.
2. 33$\frac{1}{3}$%. 5. 2%.
3. 66$\frac{2}{3}$%. 6. $200.
7. $600 coll.; $1200 sales.
8. 11$\frac{1}{9}$%.
9. 40%.
10. $6000 H. & L. $7200 Acre, P. $9600 Oil.
11. 38%.

Page 112.

12. 95%.
13. 62$\frac{2}{9}$%.
14. 18%.
15. 16$\frac{1}{2}$%.
16. 21$\frac{19}{36}$%; 84$\frac{11}{48}$%.
17. 500 sheep.
18. $1600 A., $1920 B., $1280 C.
19. 1900%.
20. 375 C., 250 P., 125 Q., 750 A.

Page 114.

Art. 337.

1. $17.50.
2. $50.
3. $250.
4. $112.50.
5. $712.50.
6. $90.
7. $40.
8. $937.50.
9. $46.67.
10. $310.
11. $11.25.
12. $4060.
13. $27.75.
14. $187.66.
15. $25.

Page 115.

Art. 339.

1. $200. 6. $4.50.
2. $300. 7. $200.
3. $875. 8. $1250.
4. $5000 9. $2250.
5. $1880. 10. $5000.
11. $2030.40.
12. $2160 butter; $2088 apples.
13. $6 per brl.
14. $800 wool, $640 hops.
15. $1000 potatoes, $900 hay.

Page 116.

Art. 341.

1. 16$\frac{2}{3}$%. 6. $\frac{1}{3}$%.
2. 15%. 7. $\frac{1}{3}$%.
3. 2%. 8. $\frac{1}{4}$%.
4. 20%. 9. 42$\frac{6}{7}$%.
5. 75%. 10. 25%.
11. 66$\frac{2}{3}$%, cost 40%.
12. 25%.
13. 20%.
14. 25%; 25%; 20%; 7$\frac{1}{2}$%.
15. 20%; 25%.

Page 117.

Art. 343.

1. $600.
2. $600.
3. $333.33.
4. $863.64.
5. $1000.
6. $5.
7. $7.20.

8. 60¢.
9. 5¢.
10. 6¾¢.

Art. 344.

1. $190.20 gain, $10\frac{200}{297}$%.
2. $500.
3. $112.50.
4. 50%.
5. $4000.
6. $600.
7. 12%.
8. $1.66⅔.
9. $3 M. $3.60 J. $4.20 R.

Page 118.

10. $4000.
11. 16⅔%. $6.
12. 62½%, $1.50, 18¾%.
13. $300 Jan. $240 Feb. $180 Mar. $180 Apr.
14. $6468.51.
15. $32800.

Page 120.

Art. 351.

1. $28.12.
2. $18562.50.
3. $1093.75.
4. $1911.88 net. $1,873.64.

Page 121.

5. $825. $783.75.

Art. 353.

1. $2.81 net cost.
2. $123.75.
3. $39.
4. $270. $225. $150. $109.38.
5. 42¢, 32¢, 18¢, 13¢, 14¢, 11¢, 9 ¢, 6¢. $2.80,$2.45,$1.75, $1.40, $1.05.

Page 123.

Art. 355.

1. $490.
2. $1032.41.
3. $29.58.
4. $342.
5. $179.20.
6. $128.25.
7. $205.20.
8. $205.20.
9. $243.
10. $1822.68.

Page 124.

Art. 357.

1. $8.21.
2. $2.27.
3. $252.
4. 35¢, 13¢, $8.64, $7.20, $5.76.
5. $7.50.

Art. 358.

1. $4650; $4557.
2. $796.35.
3. $649.66.
4. $9.72 ; $18.72 ; $34.20.
5. $400.
6. $600.

Page 125.

7. $1834.50.
8. $9.47.
9. 50%, 40%, 10%.
10. $45.
11. $6.50, 20%.
12. 66¼%, $2.70.
13. 33⅓%, 25%; $20.
14. 16⅔%.
15. $250 Barnes; $357.50 Bab ; $245 ; $350.35; $5 error.

Page 127.

Art. 366.

1. $875.05.
2. $255.70.
3. $75.08.
4. $109.50.
5. $30.05.
6. $56.25.
7. $168.07.
8. $63.04.
9. $11.88.
10. $85.57.
11. $42.53.
12. $108.97.
13. $282.09.
14. $1489.50.
15. $134.65.

Page 128.

Art. 368.

1. $1700.
2. $36000.
3. $100000.
4. $128000.
5. $7000.
6. $962.
7. $1287.50.
8. $9720.
9. $10000.
10. $7500.

Page 129.

Art. 370.

1. $1700. $51.
2. $1000. $50.
3. $200. $5.
4. $3000. $225.
5. $4000. $10.
6. $500. $50.
7. $1500. $45.
8. $700. $3.50.
9. $1200. $30.
10. $1400. $98.
11. $1478.
12. 8000 pounds.
13. $54.
14. 1360 bu.
15. $8602.94+.

Page 130.

Art. 372.

1. $532.
2. $9450.
3. $6771.25.
4. $292.46.

Page 131.

5. $5149.50.

Art. 373.

1. $594 coll. $44.55 com.
2. 1400 brls.
3. 4½%.
4. $10035.58.
5. 33⅓%.
6. 2000 lbs. $16.20 com.
7. $8179.50.

Page 132.

8. 197891.7 lbs. wool. $2019.30 com.
9. $1097.40.
10. 4½%.
11. 5%. $7830.
12. ⅛% ins. 2½% com. $3340.65 pro.
13. $1888.
14. $396 com.
15. $27155.49 remit.

Page 134.

Art. 378.

1. $17.85 bal. due.
2. $560.50.

Page 135.

Art. 379.

1. $102.81.
2. $245.89.
3. $277.79.

Page 136.

Art. 380.

1. $405. 3. $1720.
2. $17.28. 4. $10.56.
5. $34.95.

Page 137.

Art. 381.

1. $992.87.
2. $825.64 bal. $1125.64 T.
3. $205.05 bal.
4. $182.11 bal.
5. $331.30 Dr. $147.94 Cr. $183.36.

Page 138.

Art. 382.

1. $438.73.
2. $211.80.
3. $493.35.

Page 139.

4. $3558.94.
5. $27234.38.

Art. 383.

1. $176.25.
2. $75.23.
3. $124.42.
4. $796.40.
5. $122.06.

Page 140.

6. $3100.
7. $21519.75.
8. $6349.99.
9. $271.78.
10. $797.20.

Page 142.

Art. 389.

1. $1.88. 6. 33⅓¢.
2. $2.25. 7. 42¢.
3. 63¢. 8. 40¢.
4. $3.50. 9. $1.25.
5. $1.63. 10. $2.50.

Page 143.

Art. 391.

1. 19¢. 6. 73¢.
2. 26¢. 7. 91¢.
3. 36¢. 8. 92¢.
4. 46¢. 9. $1.13.
5. 55¢. 10. 63¢.

Art. 393.

1. $1.56. 6. 83¢.
2. $1. 7. 39¢.
3. $1.20. 8. $3.06.
4. $1.75. 9. $1.90.
5. $2.10. 10. $2.55.

Page 144.

Art. 395.

1. $28.
2. Hats, $20; $10; $40; $60.
Gloves, $50; $25; $75.
3. Hats, $28.12½, $18.75, $25.
Ties, $9.37½, $18.75, $28.12½.
Gloves, $15.62½, $23.43¾, $31.25.

Page 145.

Art. 397.

1. $7.50. HY.RX.
2. $36. DE.YX.
3. $48.50. OD.OX⅔.
4. $3.00. D.YG.
5. $4.50. O.YR.
6. $9.50. HA.OX⅔.
7. $3.75. S.YX.
8. $6.50. E.OX⅔.
9. $3.50. D.OX⅔.
10. $25.50. ND.YC.
11. $1. H.NX⅓.
12. 20¢. .AO⅔.
13. $2.40. N.AY.
14. $1.10. H.DO⅔.
15. $5.75. M.OX⅔.
16. 4½¢. .YO.
17. 37½¢. .SY.
18. 7½¢. .HY.
19. $5.90. M.EO⅔.
20. $6.75. L.YB.

Art. 398.

1. 46¢.
2. $1.50 V. 38¢ L. 25¢ F.
3. 90¢.

Page 146.

4. $3.15.
5. AX¢. CL¢.
6. 19¢ W. $2.13 S. C. 39¢ F. 8¢ C. 15¢ R. $4.67 S.
7. 3¢ I. 10¢ I. W. 13¢ S. 25¢ S K. $1.25 Sleds.
8. $4.50; $3.25; $2.66⅔ Boots.
$7.17, $6.17, $5.25, $4.75 Coats.
9. $3,50 P; $1.25 B; $9 P. $1.50 G. $12 T. S. $30.50 D. S. $15 B. S.
10. 18¢, 20¢, 21¢, 22¢, 23¢, 25¢, 29¢, 33¢. Hosiery.
63¢, 67¢, 68¢, 71¢, 82¢, 83¢, 88¢, 92¢. Knit Goods.

Page 149.

Art. 414.

1. $.099. 11. $.115⅙.
2. $.102. 12. $.065⅓.
3. $.208. 13. $.130½.
4. $.1085. 14. $.075⅔.
5. $.1145. 15. $.140⅚.
6. $.065⅙. 16. $.206⅙.
7. $.136⅙. 17. $.146⅓.
8. $.103⅔. 18. $.096⅔.
9. $.23. 19. $.166⅚.
10. $.064⅚. 20. $.117⅙.

Page 150.

Art. 416.

1. $53.30. 5. $78.84.
2. $23.50. 6. $113.75.
3. $44.89. 7. $348.67.
4. $62.23. 8. $1155.
9. $935.
10. $1697.40.
11. $27.50. 12. $63.
13. $116.25.
14. $194.75.
15. $2.75. 17. $11.63.
16. $8.40. 18. $20.09.
19. $902.60.
20. $120.61.

Page 151.

Art. 418.

1. $51.19. 2. $119.88.

3. $72.71. 5. $157.20.
4. $185. 6. $204.38.
7. $1481.58.
8. $86.35. 9. $20.27.
10. $182.42.

Page 152.

Art. 420.

1. $2. 10. $1.35.
2. $8.40. 11. 42¢.
3. 75¢. 12. $2.32.
4. $7.50. 13. $5.94.
5. $13.50. 14. $4.43.
6. $6.53. 15. $14.67.
7. 88¢. 16. $11.19.
8. 70¢. 17. $17.50.
9. 10¢. 18. $30.17.
19. $25.
20. $170.83.

Page 153.

Art. 422.

1. $40. 8. $7.25.
2. $8.70. 9. $14.06.
3. $4.80. 10. $13.14.
4. $3.75. 11. $7.25.
5. $4.07. 12. $3.78.
6. $14.20. 13. $6.38.
7. $8.35. 14. $.14.
15. $.39.

Art. 423.

1. 16.1 ms. 11. 27.1$\frac{1}{3}$.
2. 15.2. 12. 46.1$\frac{2}{3}$.
3. 17.3. 13. 21.2$\frac{1}{3}$.
4. 22.4. 14. 23.2$\frac{2}{3}$.
5. 20.5. 15. 18.3$\frac{1}{3}$.
6. 13.6. 16. 16.3$\frac{2}{3}$.
7. 23.7. 17. 21.9$\frac{2}{3}$.
8. 25.8. 18. 46.9$\frac{1}{3}$.
9. 39.9. 19. 13.0$\frac{1}{3}$.
10. 41.0$\frac{1}{3}$. 20. 23.9$\frac{1}{3}$.

Page 154.

Art. 424.

1. $143.60. 5. $256.02.
2. $41.75. 6. $557.77.
3. $1730. 7. $255.80.
4. $783.75. 8. $210.64.
9. $143.12.
10. $1704.23.
11. $957.87.
12. $122.91. 13. $112.
14. $206.55.
15. $301.82.

Page 155.

Art. 426.

1. $900. 2. $629.20.
3. $900.
4. $1015.50.
5. $9742.86. 8. $1400.
6. $1200. 9. $1800.
7. $1200. 10. $1640.

Art. 428.

1. 6%. 6. 6%.
2. 6%. 7. 6%.
3. 8%. 8. 6%.
4. 3%. 9. 9%.
5. 2$\frac{1}{2}$%. 10. 9%.

Page 156.

Art. 430.

1. 6 mos. 5. 15 ds.
2. 2 yrs. 6. 6 ds.
3. 21 ds. 7. 45 ds.
4. 3 mos. 8. 1 yr.
9. 2 yr., 7 mos. 24 ds.
10. 3 mos., 10 ds.

Page 157.

Art. 432.

1. $300. 6. $400.
2. $500. 7. $850.
3. $800. 8. $50.
4. $644.73. 9. $150.
5. $300. 10. $15000.

Page 158.

Art. 434.

1. $11.53. 6. $3.95.
2. $3.62. 7. $5.10.
3. $2.80. 8. $16.95.
4. $5.19. 9. $4.50.
5. $16.81. 10. $2.59.

Page 159.

Art. 439.

1. $448.70.
2. $1546.70.

Page 160.

3. $366.60.
4. $2422.29.
5. $12726.80.

Page 162.

Art. 444.

1. $314.97. 3. $75.20.
2. $95.82. 4. $417.84.
5. $1285.81.
6. $1775.25.

Art. 445.

1. $5.14. 2. $1064.16.
3. 16$\frac{2}{3}$ yrs.; 20 yrs.; 25 yrs.
4. $933.64. 5. 6%.
6. 3 yrs., 7 mos., 6 ds.

Page 163.

7. $4620. 8. 725 M.
9. $1756.27.
10. $298.86.
11. 4$\frac{13}{18}$%. 12. $1361.07.
13. May 18, '89.
14. $875 gain.

Page 164.

15. $2728.82.
16. $536.95 better in land.
17. $3755.33.
18. $2854.73.
19. $30000.
20. 1 yr., 8 mos., 12 ds. 3 yr., 4 mos., 24 ds.

Page 166.

Art. 449.

1. $540.
2. $100. $38.85 dis.
3. $1654.61.
4. $12.29 Int. greater.
5. 55¢ better at $6, cash.
6. No difference.
7. $11.82 loss.
8. $1523.15.
9. $7.48. 10. 4% loss.
11. First $8.37 better.
12. 21$\frac{333}{427}$% profit.

Page 168.

Art. 457.

1. $18.60 dis.; $1181.40 pro.
2. $10.18 dis.; $1839.82 pro.
3. $27.37.
4. $4897.50.
5. $18.30.
6. Aug. 4; 95 ds.; $399.35 pro.

Page 169.

7. $1.80.
8. $450. No proceeds.
9. $161.14.
10. $4.53 dis.; $499.35 proceeds.

Page 170.

Art. 459.

1. $502.78.
2. $1500. 3. $500.
4. $253.94.
5. $8000.

Page 171.

Art. 461.

1. $603.32.
2. $1515.92.
3. $6033.18.
4. $408.37.
5. $507.87.
6. $252.65.
7. $452.91.
8. $910.58.
9. $760.65.
10. $1209.98.

Art. 463.

1. $880.42.
2. $750.21.
3. $4950.
4. $5970.50.
5. $9420.
6. $7835.
7. $741.18.
8. $1733.37.
9. $8357.62.
10. $1590.67.

Page 172.

Art. 465.

1. Feb. 3.
2. July 12.
3. Aug. 1.
4. June 2.
5. May 13.
6. May 18.
7. May 16.
8. Apr. 28.
9. June 21.
10. July 4.

Page 173.

Art. 467.

1. $6.83 dis.; $643.17 proceeds.
2. May 15; $3.20 dis. $396.80 pro.
3. $640.36 proceeds.
4. July 18; $78.69; $4998.19.

Page 174.

5. $100.
6. $5000.58.
7. $74143.75.
8. $812.59.
9. $857.52.
10. 874; 53¢ dis.; $399.47 proceeds
875; $6.30 dis.; $600 face.
876; $68333.33; $68230.83.
877; 33 ds.; $^5/_{28}$
878; $1500 pro.; $15.75 dis.
879; $124 dis.
880; $6334.50; face. $6292.27 pro.

Page 177.

Art. 472.

1. $340.66.
2. $735.53.

Page 178.

3. $656.82.
4. $50.95.
5. $695.43.
6. $344.65.

Page 180.

Art. 474.

1. $522.91.
2. $44.88.
3. $293.51.
4. $640.89.
5. $559.97.

Page 183.

Art. 482.

1. Feb. 10. *3.* Mar. 2.
2. July 11. *4.* Mar. 9.

Page 184.

5. Jan. 21. *6.* Apr. 5.

Page 185.

Art. 484.

1. May 30. *3.* July 11.
2. Mar. 28. *4.* Aug. 29.

Page 186.

5. Apr. 7.
6. May 9.

Page 187.

Art. 486.

1. Feb. 7.
2. Apr. 30.
3. Mar. 4.

Page 188.

4. $5140; Aug. 15.
5. $154.38; $8.24; Jan. 15, '91.
6. Jan. 25, '91; $369.89; $355.73.
7. $410. July 24, '90.

Page 189.

8. $150; Mar. 12, '91.
9. $292.25: $295.09; Feb. 16, '91.
10. Dec. 5.

Page 191.

Art. 496.

1. $5645.50.
2. $1744.80.

Page 192.

3. $1178.80.
4. $2188.60.
5. $904.70.
6. $686.35.
7. $900.90.
8. $3565.68.
9. $757.20.
10. $10531.70.
11. $1970.90.
12. $4562.50.

Page 193.

Art. 503.

1. $273.75.

Page 194.

2. $377.81. *4.* $522.50.
3. $78.13. *5.* $180.47.

Page 195.

Art. 504.

1. $\frac{5}{8}$%; $62.50.
2. $3550.
3. $537.50.
4. $4\frac{1}{2}$ mills. $68.13.
5. $3\frac{1}{4}$ mills; $88883.13, $27553.77 school tax.

Page 197.

Art. 523.

1. $11.81.

Page 198.

2. $6440. *4.* $4.
3. $5000. *5.* $120.
6. $343.75.
7. $380000.
8. $120000.
9. $37500.
10. $9720 Ætna.
$8100 Hartford.
$6480 Phœnix.
$8100 Man.
$8100 Prov.

Page 201.

Art. 538.

1. $9000 div. $54 B.
2. $96250 div. $44 A.
3. $22500 div. $93.75 Bishop.

Page 202.

4. $2625 div. $87.50 C
5. $855.38 sinking.
   $16000 div.
   $11657.87 undivided profits.

Page 203.

Art. 540.

1. 8%. 3. $3\frac{1}{2}$%.
2. 18%. 4. 8%. $850.
5. 9%. $1701 undivided profits.

Page 204.

Art. 542.

1. $16000. 6. $15000.
2. $25000. 7. $72500.
3. $17000. 8. $16000.
4. $89800. 9. $33800.
5. $19500. 10. $21600.

Art. 544.

1. $4637.50.
2. $2362.50.
3. $6030. 6. $3625.
4. $4350. 7. $12750.
5. $14400. 8. $21900.
9. $77062.50.
10. $50625.

Page 205.

Art. 546.

1. $16\frac{2}{3}$%. 6. 20%.
2. $11\frac{1}{9}$%. 7. $12\frac{1}{2}$%.
3. 30%. 8. $6\frac{6}{19}$%.
4. $16\frac{2}{3}$%. 9. $5\frac{5}{24}$%.
5. $30\frac{10}{13}$%. 10. $12\frac{1}{2}$%.

Art. 547.

1. 5%. 3. $255.
2. $6\frac{1}{2}$%. 4. $11456.25.
5. 120 shares.
6. $28850.01.
7. 125%. Brok. $100.
8. $8250.

Page 212.

Art. 565.

1. $2940.
2. $1948.65.

Page 213.

3. $38.37.
4. $2582.58.
5. $81239.91.
6. $3.13.
7. $3756.30. $43.70.
8. $5080.98. $344.42.
9. $10564.83.

Page 216.

Art. 576.

1. $576 Pritchard.
   $864 Gaines.
2. $466.67 A.
   $350 B.
   $233.33 C.
3. $337.50 Jones.
   $562.50 Smalley.
   $600 Parker.
4. $2000 Gould.
   $3000 Brooks }
   $5000 Lansing } Net Gain.
   $23500 Guild }
   $34000 Brooks }
   $55000 Lansing. P. W.
5. $8880 Mar. Gain.
   $31080 Mar. P. W.
   $5920 Bant. Gain.
   $20920 Bant. P.W.

Page 218.

Art. 578.

1. $150 A.
   $250 B.
2. $80 A.
   $100 B.
   $120 C.
3. $52.50 A.
   $14 B.
   $84 C.
4. $140 Moore.
   $112 Parks.
5. $171 Ensign.
   $120 Gould.
6. $139.20 Brown.
   $153.70 Smith.

Page 221.

Art. 580.

1. $3224 Gain of each.
   $24024 DeV. P.W.
   $24224 VanS. P. W.
2. $2225 Gain of each.
   $12225 Ham. P.W.
   $12825 Perr. P. W.
3. $6125 Gain of each.
   $33425 Hayes P.W.
   $23025 McN. P.W.
4. $1825 Gain of each.
   $7425 West. P. W.
   $7825 Far. P. W.
5. $28571 L. C. Guiness P. W.
   $28571 J. L. Guiness P. W.
   $40471 M. N. Porter P. W.

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Art. 581.

1. $4266.67 A.
   $5333.33 B.
   $6400 C.
2. $185.53 A.
   $278.30 B.
   $231.92 C.
3. $252 Leroy.
   $198 Huestis.
4. $2000 Gain of each.
5. $11490.91 Larkin.
   $14354.54 Pope.
   $19754.55 Henry.

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6. $483.45 Gain of ea.
   $4183.45 Allen P. W.
   $4883.45 Furbeck P. W.
7. $660 Foreman.
   $2640 Blanchard.
8. $175 Gain of each.
   $10775 H. L. McD. P. W.
   $675 J. W. McD. P. W.

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9. $3750 Gain of each.
   $620 Sprague P. A.
   $839 Mareniss P.A.
   $8870 Spra. P. W.
   $15439 Mar. P. W.
10. $7122.27 Net Gain.
   $2374.09 sh. of ea.
   $8414.09 Morris.
   $9174.09 Perkins.
   $9734.09 Livingst'n
   $27322.27 Net cap.